Government of Karnataka

GANITA PRAKASH

Textbook of Mathematics

6

SIXTH STANDARD
Part - 2

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एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

Karnataka Textbook Society (R.)
6th Cross, Malleshwaram, Bengaluru-560 003

Constitution of India

Part IV A (Article 51 A)

Fundamental Duties

It shall be the duty of every citizen of India —

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers, wildlife and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

*(k) who is a parent or guardian, to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

Note: The Article 51A containing Fundamental Duties was inserted by the Constitution (42nd Amendment) Act, 1976 (with effect from 3 January 1977).

*(k) was inserted by the Constitution (86th Amendment) Act, 2002 (with effect from 1 April 2010).

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CONSTITUTION OF INDIA

Part III (Articles 12 – 35)
(Subject to certain conditions, some exceptions and reasonable restrictions)

guarantees these

Fundamental Rights

Right to Equality

- before law and equal protection of laws;
- irrespective of religion, race, caste, sex or place of birth;
- of opportunity in public employment;
- by abolition of untouchability and titles.

Right to Freedom

- of expression, assembly, association, movement, residence and profession;
- of certain protections in respect of conviction for offences;
- of protection of life and personal liberty;
- of free and compulsory education for children between the age of six and fourteen years;
- of protection against arrest and detention in certain cases.

Right against Exploitation

- for prohibition of traffic in human beings and forced labour;
- for prohibition of employment of children in hazardous jobs.

Right to Freedom of Religion

- freedom of conscience and free profession, practice and propagation of religion;
- freedom to manage religious affairs;
- freedom as to payment of taxes for promotion of any particular religion;
- freedom as to attendance at religious instruction or religious worship in educational institutions wholly maintained by the State.

Cultural and Educational Rights

- for protection of interests of minorities to conserve their language, script and culture;
- for minorities to establish and administer educational institutions of their choice.

Right to Constitutional Remedies

- by issuance of directions or orders or writs by the Supreme Court and High Courts for enforcement of these Fundamental Rights.

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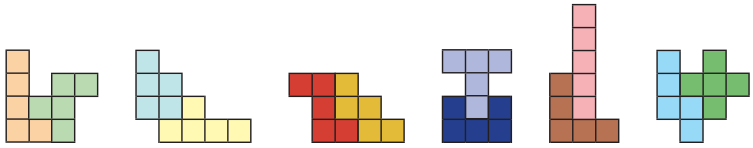
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6 PERIMETER AND AREA

6.1 Perimeter

Do you remember what the perimeter of a closed plane figure is? Let us refresh our understanding!

The perimeter of any closed plane figure is the distance covered along its boundary when you go around it once. For a **polygon**, i.e., a closed plane figure made up of line segments, the perimeter is simply the sum of the lengths of its all sides, i.e., the total distance along its outer boundary.

The perimeter of a polygon = the sum of the lengths of its all sides.

Let us revise the formulas for the perimeter of rectangles, squares, and triangles.

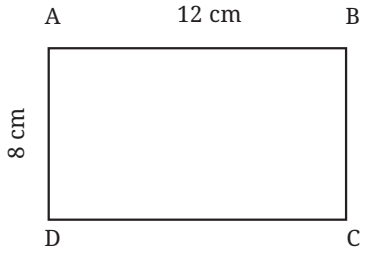
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Perimeter of a rectangle

Consider a rectangle ABCD whose length and breadth are 12 cm and 8 cm, respectively. What is its perimeter?

Perimeter of the rectangle = Sum of the lengths of its four sides

= AB + BC + CD + DA



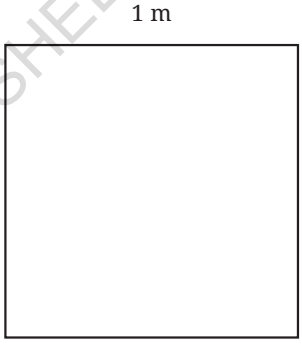
= AB + BC + AB + BC
= 2 × AB + 2 × BC
= 2 × (AB + BC)
= 2 × (12 cm + 8 cm)
= 2 × (20 cm)
= 40 cm.

Opposite sides of a rectangle are always equal. So, AB = CD and AD = BC

From this example, we see that—
Perimeter of a rectangle = length + breadth + length + breadth.
Perimeter of a rectangle = 2 × (length + breadth).
The perimeter of a rectangle is twice the sum of its length and breadth.

Perimeter of a square

Debojeet wants to put coloured tape all around a square photo frame of side 1m as shown. What will be the length of the coloured tape he requires? Since Debojeet wants to put the coloured tape all around the square photo frame, he needs to find the perimeter of the photo frame.



Thus, the length of the tape required = perimeter of the square
= sum of the lengths of all four sides of the square
= 1 m + 1 m + 1 m + 1 m = 4 m.

Now, we know that all four sides of a square are equal in length. Therefore, in place of adding the lengths of each side, we can simply multiply the length of one side by 4.

Thus, the length of the tape required = 4 × 1 m = 4 m.

From this example, we see that

Perimeter of a square = 4 × length of a side.

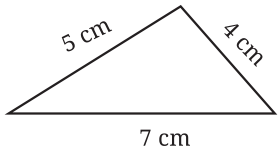
The perimeter of a square is quadruple the length of its side.

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Perimeter of a Triangle

Consider a triangle having three given sides of lengths 4 cm, 5 cm and 7 cm. Find its perimeter.



Perimeter of the triangle = 4 cm + 5 cm + 7 cm
= 16 cm.

Perimeter of a triangle = sum of the lengths of its three sides.

Example: Akshi wants to put lace all around a rectangular tablecloth that is 3 m long and 2 m wide. Find the length of the lace required.



Solution

Length of the rectangular table cover = 3 m.

Breadth of the rectangular table cover = 2 m.

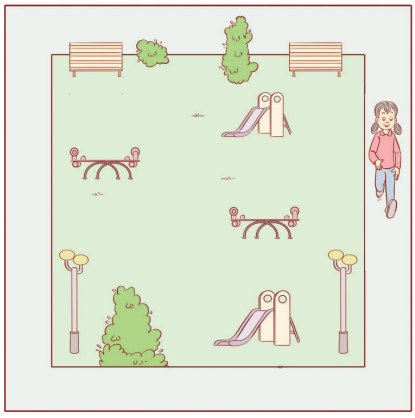
Akshi wants to put lace all around the tablecloth.

Therefore, the length of the lace required will be the perimeter of the rectangular tablecloth.

Now, the perimeter of the rectangular tablecloth = 2 × (length + breadth)
= 2 × (3 m + 2 m) = 2 × 5 m = 10 m.

Hence, the length of the lace required is 10 m.

Example: Find the distance travelled by Usha if she takes three rounds of a square park of side 75 m.



Solution

Perimeter of the square park = 4 × length of a side = 4 × 75 m = 300 m.

Distance covered by Usha in one round = 300 m.

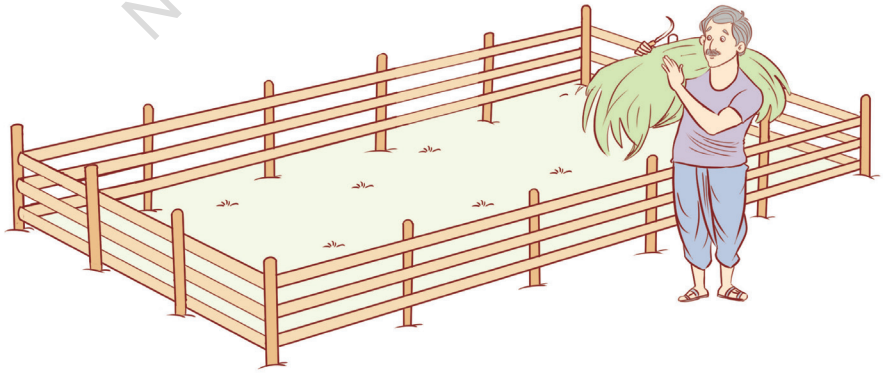
Therefore, the total distance travelled by Usha in three rounds = 3 × 300 m = 900 m.

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Figure it Out

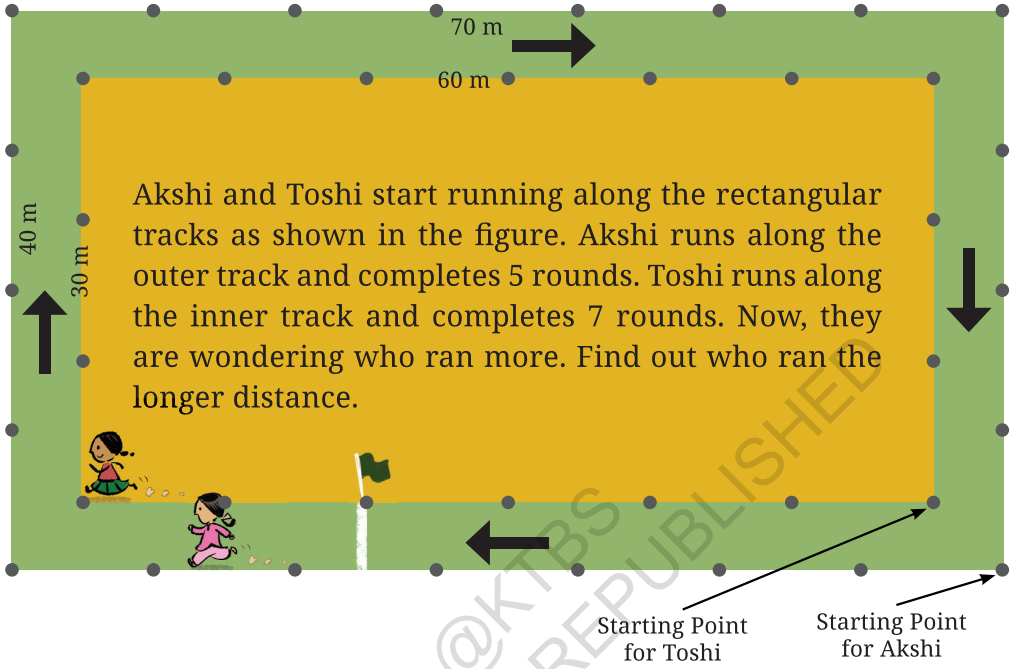
1. Find the missing terms:
   a. Perimeter of a rectangle = 14 cm; breadth = 2 cm; length = ?.
   b. Perimeter of a square = 20 cm; side of a length = ?.
   c. Perimeter of a rectangle = 12 m; length = 3 m; breadth = ?.
2. A rectangle having sidelengths 5 cm and 3 cm is made using a piece of wire. If the wire is straightened and then bent to form a square, what will be the length of a side of the square?
3. Find the length of the third side of a triangle having a perimeter of 55 cm and having two sides of length 20 cm and 14 cm, respectively.
4. What would be the cost of fencing a rectangular park whose length is 150 m and breadth is 120 m, if the fence costs ₹40 per metre?
5. A piece of string is 36 cm long. What will be the length of each side, if it is used to form:
   a. A square,
   b. A triangle with all sides of equal length, and
   c. A hexagon (a six sided closed figure) with sides of equal length?
6. A farmer has a rectangular field having length 230 m and breadth 160 m. He wants to fence it with 3 rounds of rope as shown. What is the total length of rope needed?



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Matha Pachchi!



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Each track is a rectangle. Akshi's track has length 70 m and breadth 40 m. Running one complete round on this track would cover 220 m, i.e., 2 × (70 + 40) m = 220 m. This is the distance covered by Akshi in one round.

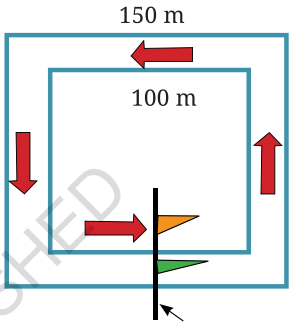
Figure it Out

1. Find out the total distance Akshi has covered in 5 rounds.
2. Find out the total distance Toshi has covered in 7 rounds. Who ran a longer distance?
3. Think and mark the positions as directed—
   a. Mark 'A' at the point where Akshi will be after she ran 250 m.
   b. Mark 'B' at the point where Akshi will be after she ran 500 m.
   c. Now, Akshi ran 1000 m. How many full rounds has she finished running around her track? Mark her position as 'C'.
   d. Mark 'X' at the point where Toshi will be after she ran 250 m.
   e. Mark 'Y' at the point where Toshi will be after she ran 500 m.

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f. Now, Toshi ran 1000 m. How many full rounds has she finished running around her track? Mark her position as 'Z'.

Deep Dive: In races, usually there is a common finish line for all the runners. Here are two square running tracks with the inner track of 100 m each side and outer track of 150 m each side. The common finishing line for both runners is shown by the flags in the figure which are in the center of one of the sides of the tracks.



If the total race is of 350 m, then we have to find out where the starting positions of the two runners should be on these two tracks so that they both have a common finishing line after they run for 350 m. Mark the starting points of the runner on the inner track as 'A' and the runner on the outer track as 'B'.

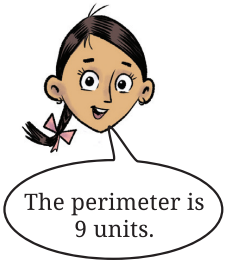
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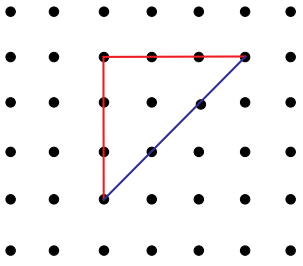
Estimate and Verify



Take a rough sheet of paper or a sheet of newspaper. Make a few random shapes by cutting the paper in different ways. Estimate the total length of the boundaries of each shape then use a scale or measuring tape to measure and verify the perimeter for each shape.

Akshi says that the perimeter of this triangle shape is 9 units. Toshi says it can't be 9 units and the perimeter will be more than 9 units. What do you think?



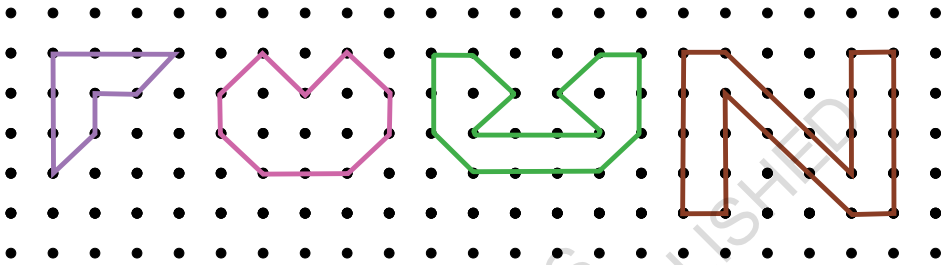




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This figure has lines of two different unit lengths. Measure the lengths of a red line and a blue line; are they same? We will call the red lines—straight lines and the blue lines—diagonal lines. So, the perimeter of this triangle is 6 straight units + 3 diagonal units. We can write this in a short form as: 6s + 3d units.

Write the perimeters of the figures below in terms of straight and diagonal units.



Perimeter of a regular polygon

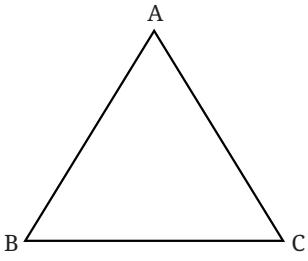
Like squares, closed figures that have all sides and all angles equal are called **regular polygons**. We studied the sequence of regular polygons as 'Shape Sequence' #1 in Chapter 1. Examples of regular polygons are the equilateral triangle (where all three sides and all three angles are equal), regular pentagon (where all five sides and all five angles are equal), etc.

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Perimeter of an equilateral triangle

We know that for any triangle its perimeter is sum of all three sides.

Using this understanding, we can find the perimeter of an equilateral triangle.



Perimeter of an equilateral triangle
= AB + BC + AC = AB + AB + AB
= 3 times length of one side.
Perimeter of an equilateral triangle = 3 × length of a side.

What is a similarity between a square and an equilateral triangle?

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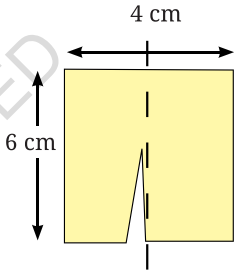
Find various objects from your surroundings that have regular shapes and find their perimeters. Also, generalise your understanding for the perimeter of other regular polygons.

Teacher's Note

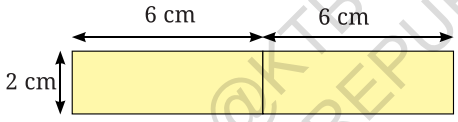
Discuss more about regular polygons and encourage students to come up with a general formula for the perimeter of a regular polygon.

Split and rejoin

A rectangular paper chit of dimension 6 cm × 4 cm is cut as shown into two equal pieces. These two pieces are joined in different ways.



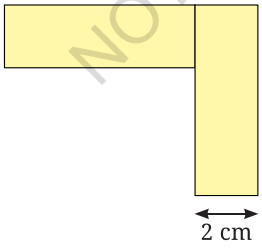
a.



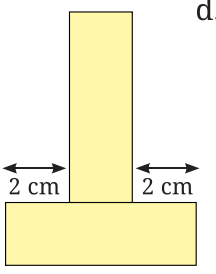
For example, the arrangement a. has a perimeter of 28 cm.

Find out the length of the boundary (i.e., the perimeter) of each of the other arrangements below.

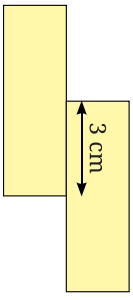
b.



c.



d.



Arrange the two pieces to form a figure with a perimeter of 22 cm.

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6.2 Area

We have studied the areas of closed figures (regular and irregular) in previous grades. Let us recall some key points.

The amount of region enclosed by a closed figure is called its **area**.

In previous grades, we arrived at the formula for the area of a rectangle and a square using square grid paper. Do you remember?

Area of a square = _____

Area of a rectangle = _____

Teacher's Note

Help students in recalling the method of finding the area of a rectangle and a square using grid papers. Provide square grid papers to students and let them come up with the formula.

Let's see some real-life problems related to these ideas.

Example: A floor is 5 m long and 4 m wide. A square carpet of sides 3 m is laid on the floor. Find the area of the floor that is not carpeted.

Solution

Length of the floor = 5 m.

Width of the floor = 4 m.

Area of the floor = length × width = 5 m × 4 m = 20 sq m.

Length of the square carpet = 3 m.

Area of the carpet = length × length = 3 m × 3 m = 9 sq m.

Hence, the area of the floor laid with carpet is 9 sq m.

Therefore, the area of the floor that is not carpeted is: area of the floor minus the area of the floor laid with carpet = 20 sq m – 9 sq m = 11 sq m.

Example: Four square flower beds each of side 4 m are in four corners on a piece of land 12 m long and 10 m wide. Find the area of the remaining part of the land.

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Solution

Length of the land (l) = 12 m.

Width of land (w) = 10 m.

Area of the whole land = $l \times w$ = 12 m × 10 m = 120 sq m.

The sidelength of each of the four square flower beds is (s) = 4 m.

Area of one flower bed = $s \times s$ = 4 m × 4 m = 16 sq m.

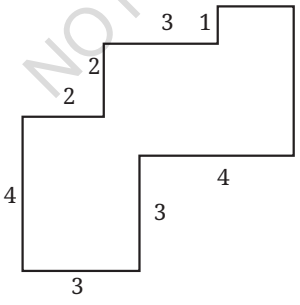
Hence, the area of the four flower beds = 4 × 16 sq m = 64 sq m.

Therefore, the area of the remaining part of the land is: area of the complete land minus the area of all four flower beds = 120 sq m – 64 sq m = 56 sq m.

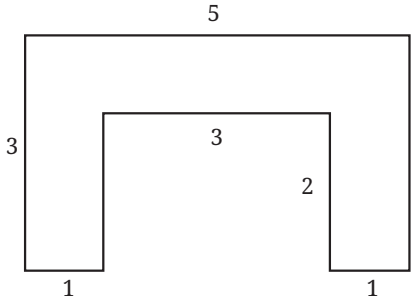
Figure it Out

1. The area of a rectangular garden 25 m long is 300 sq m. What is the width of the garden?
2. What is the cost of tiling a rectangular plot of land 500 m long and 200 m wide at the rate of ₹8 per hundred sq m?
3. A rectangular coconut grove is 100 m long and 50 m wide. If each coconut tree requires 25 sq m, what is the maximum number of trees that can be planted in this grove?
4. By splitting the following figures into rectangles, find their areas (all measures are given in metres).

a.



b.

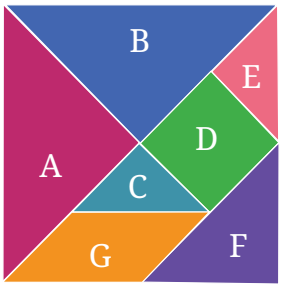


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Figure it Out

Cut out the tangram pieces given at the end of your textbook.



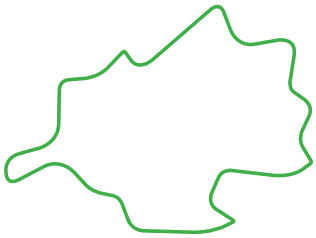
1. Explore and figure out how many pieces have the same area.
2. How many times bigger is Shape D as compared to Shape C? What is the relationship between Shapes C, D and E?
3. Which shape has more area: Shape D or F? Give reasons for your answer.
4. Which shape has more area: Shape F or G? Give reasons for your answer.
5. What is the area of Shape A as compared to Shape G? Is it twice as big? Four times as big?

   Hint: In the tangram pieces, by placing the shapes over each other, we can find out that Shapes A and B have the same area, Shapes C and E have the same area. You would have also figured out that Shape D can be exactly covered using Shapes C and E, which means Shape D has twice the area of Shape C or shape E, etc.
6. Can you now figure out the area of the big square formed with all seven pieces in terms of the area of Shape C?
7. Arrange these 7 pieces to form a rectangle. What will be the area of this rectangle in terms of the area of Shape C now? Give reasons for your answer.
8. Are the perimeters of the square and the rectangle formed from these 7 pieces different or the same? Give an explanation for your answer.

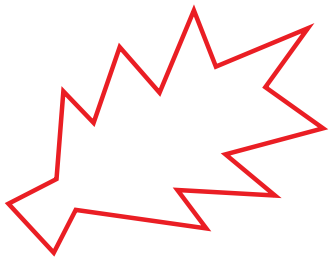
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Look at the figures below and guess which one of them has a larger area.



a.



b.

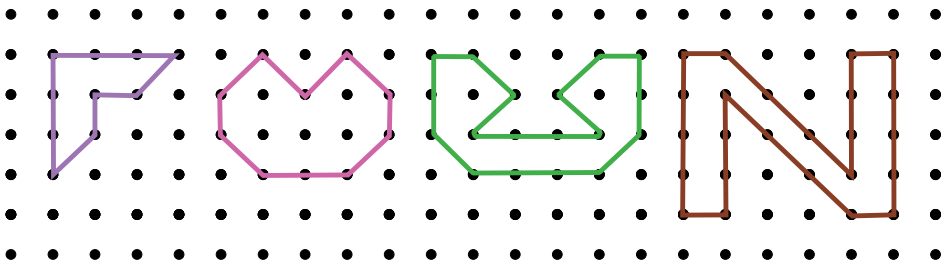
We can estimate the area of any simple closed shape by using a sheet of squared paper or graph paper where every square measures 1 unit × 1 unit or 1 square unit.

To estimate the area, we can trace the shape onto a piece of transparent paper and place the same on a piece of squared or graph paper and then follow the below conventions—

1. The area of one full small square of the squared or graph paper is taken as 1 sq unit.
2. Ignore portions of the area that are less than half a square.
3. If more than half of a square is in a region, just count it as 1 sq unit.
4. If exactly half the square is counted, take its area as $\frac{1}{2}$ sq unit.

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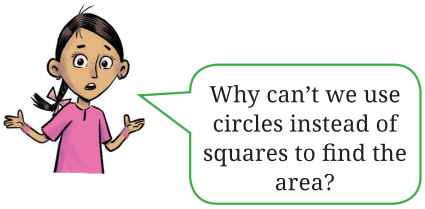
Find the area of the following figures.



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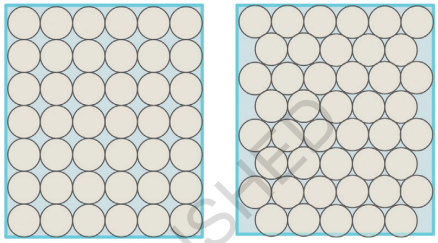
Let's Explore!

Why is area generally measured using squares?



Draw a circle on a graph sheet with diameter (breadth) of length 3. Count the squares and use them to estimate the area of the circular region.

As you can see, circles can't be packed tightly without gaps in between. So, it is difficult to get an accurate measurement of area using circles as units. Here, the same rectangle is packed in two different ways with circles—the first one has 42 circles and the second one has 44 circles.



Try using different shapes (triangle and rectangle) to fill the given space (without overlaps and gaps) and find out the merits associated with using a square shape to find the area rather than another shape. List out the points that make a square the best shape to use to measure area.

1. Find the area (in square metres) of the floor outside of the corridor.
2. Find the area (in square metres) occupied by your school playground.

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Let's Explore!

On a squared grid paper (1 square = 1 square unit), make as many rectangles as you can whose lengths and widths are a whole number of units such that the area of the rectangle is 24 square units.

Math Talk

a. Which rectangle has the greatest perimeter?

b. Which rectangle has the least perimeter?

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c. If you take a rectangle of area 32 sq cm, what will your answers be? Given any area, is it possible to predict the shape of the rectangle with the greatest perimeter as well as the least perimeter? Give examples and reasons for your answer.

6.3 Area of a Triangle

Draw a rectangle on a piece of paper and draw one of its diagonals. Cut the rectangle along that diagonal and get two triangles.

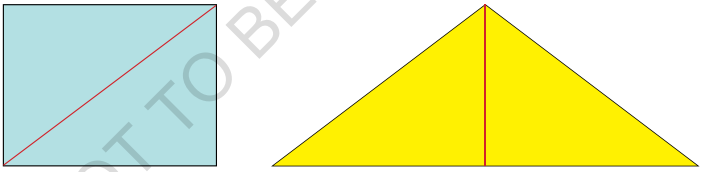
Check! whether the two triangles overlap each other exactly. Do they have the same area?

Try this with more rectangles having different dimensions. You can check this for a square as well.

Can you draw any inferences from this exercise? Please write it here. _____

_____

Now, see the figures below. Is the area of the blue rectangle more or less than the area of the yellow triangle? Or is it the same? Why?



Can you see some relationship between the blue rectangle and the yellow triangle and their areas? Write the relationship here.

_____

Teacher's Note

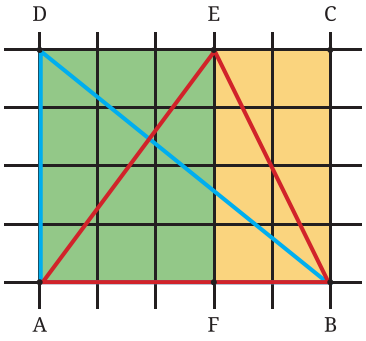
Help students in articulating their inferences and in defining the relationships they have observed in their own words, gradually leading to a common statement for whole classroom. Recall the definition of a diagonal in the classroom.

Draw suitable triangles on grid paper to verify your inferences and relationships observed in the above exercises.

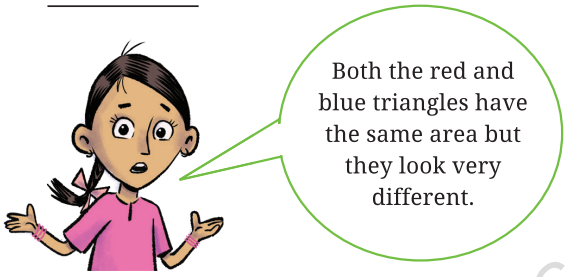
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Use your understanding from previous grades to calculate the area of any closed figure using grid paper and—

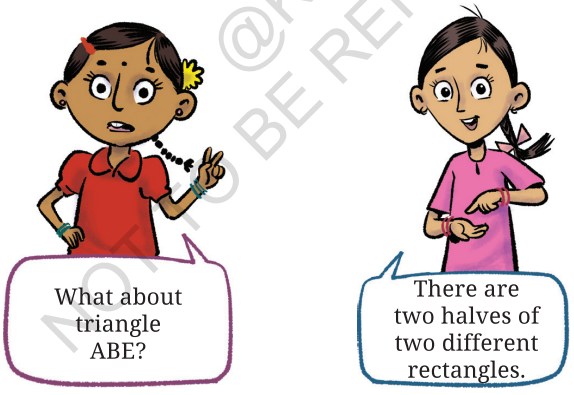


1. Find the area of blue triangle BAD. _____
2. Find the area of red triangle ABE. _____



Area of rectangle ABCD = _____

So, the area of triangle BAD is half of the area of the rectangle ABCD.



Area of triangle ABE = Area of triangle AEF + Area of triangle BEF.

Here, the area of triangle AEF = half of the area of rectangle AFED.

Similarly, the area of triangle BEF = half of the area of rectangle BFEC.

Thus, the area of triangle ABE = half of the area of rectangle AFED + half of the area of rectangle BFEC

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= half of the sum of the areas of the rectangles AFED and BFEC

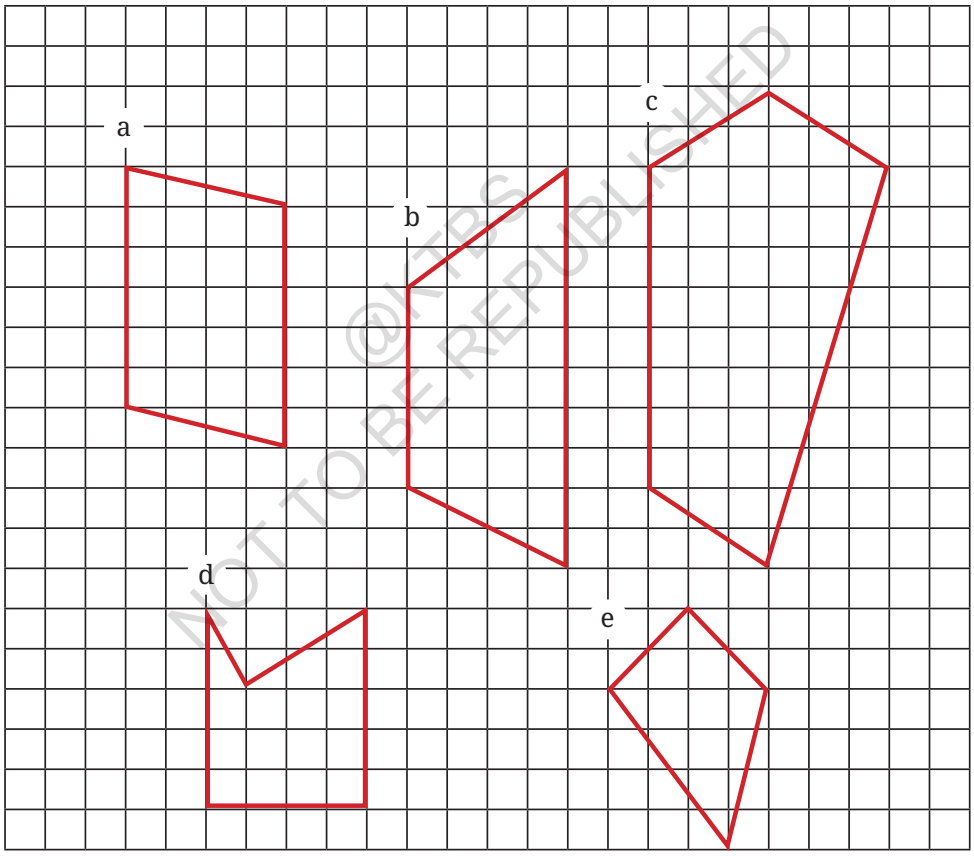
= half of the area of rectangle ABCD.

Conclusion _____

_____

Figure it Out

1. Find the areas of the figures below by dividing them into rectangles and triangles.

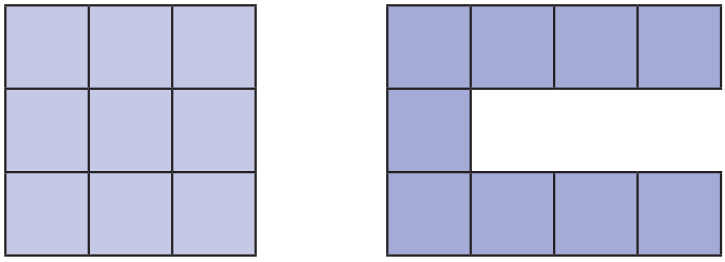


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Making it 'More' or 'Less'

Observe these two figures. Is there any similarity or difference between the two?



Using 9 unit squares (having an area of 9 sq units), we have made figures with two different perimeters—the first figure has a perimeter of 12 units and the second has a perimeter of 20 units.

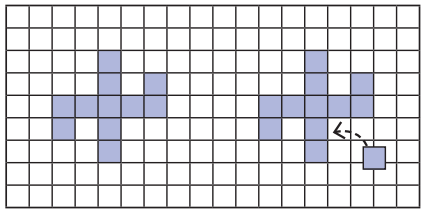
Arrange or draw different figures with 9 sq units to get other perimeters. Each square should align with at least one other square on at least one side completely and together all squares should form a single connected figure with no holes.

Using 9 unit squares, solve the following.

1. What is the smallest perimeter possible?
2. What is the largest perimeter possible?
3. Make a figure with a perimeter of 18 units.
4. Can you make other shaped figures for each of the above three perimeters, or is there only one shape with that perimeter? What is your reasoning?

Let's do something tricky now! We have a figure below having perimeter 24 units.

Without calculating all over again, observe, think and find out what will be the change in the perimeter if a new square is attached as shown on the right.

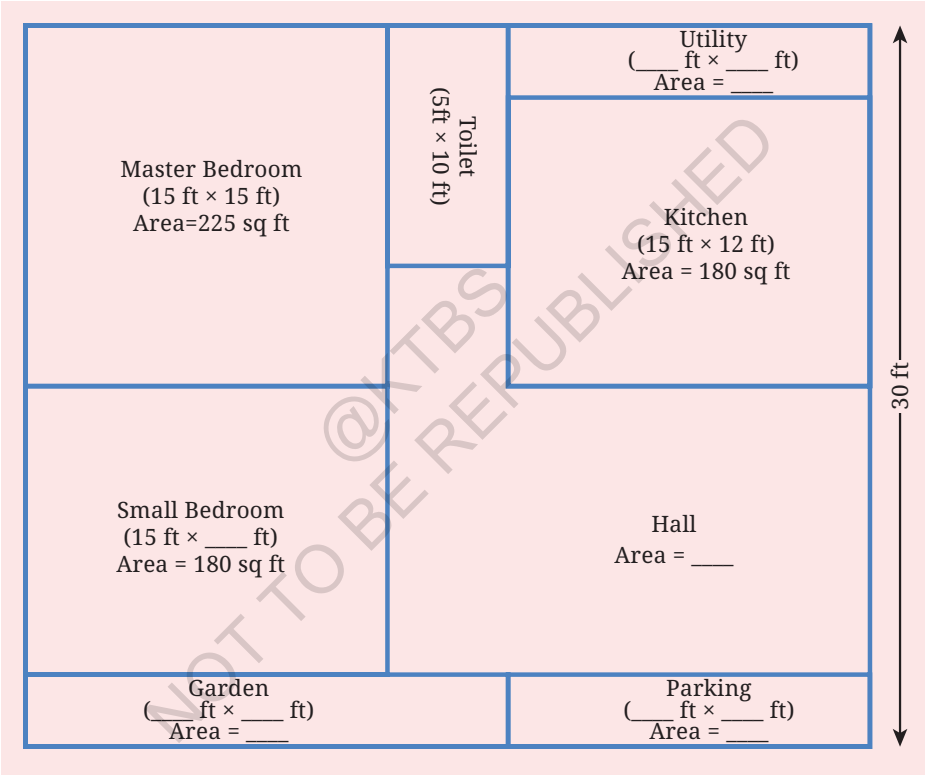


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Experiment placing this new square at different places and think what the change in perimeter will be. Can you place the square so that the perimeter: a) increases; b) decreases; c) stays the same?

Below is the house plan of Charan. It is in a rectangular plot. Look at the plan. What do you notice?



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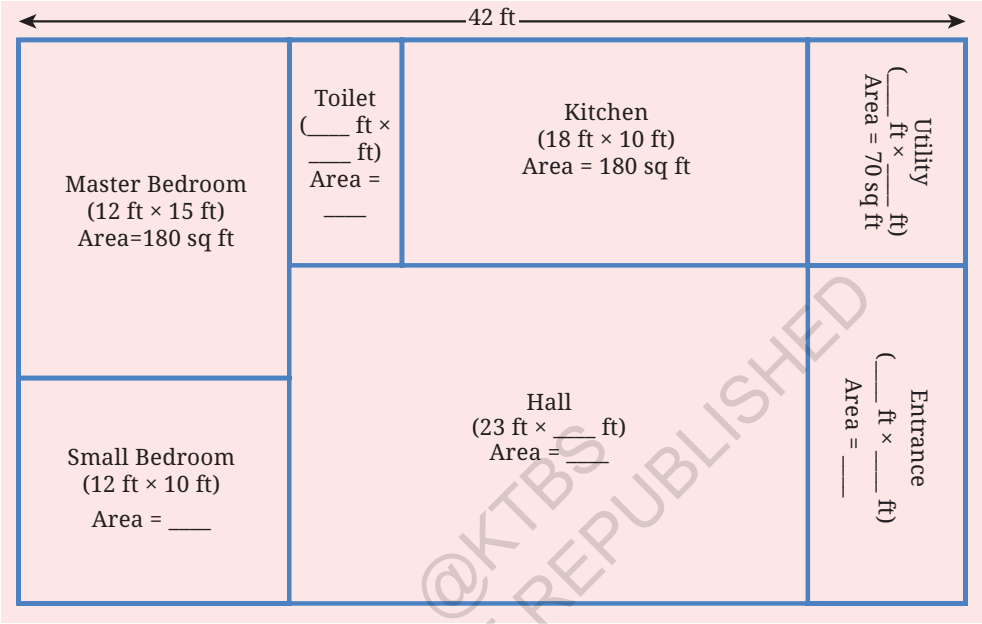
Some of the measurements are given.

a. Find the missing measurements.

b. Find out the area of his house.

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Now, find out the missing dimensions and area of Sharan's home. Below is the plan:



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Some of the measurements are given.

a. Find the missing measurements.

b. Find out the area of his house.

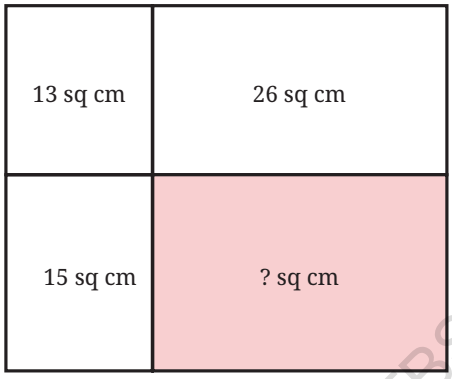
What are the dimensions of all the different rooms in Sharan's house? Compare the areas and perimeters of Sharan's house and Charan's house.

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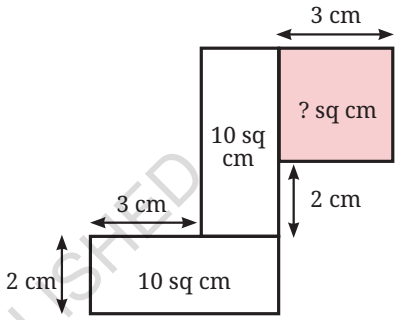
Area Maze Puzzles

In each figure, find the missing value of either the length of a side or the area of a region.

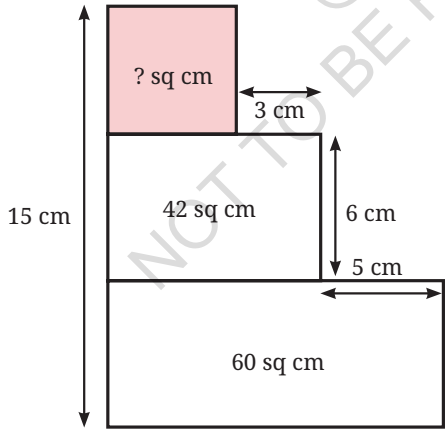
a.



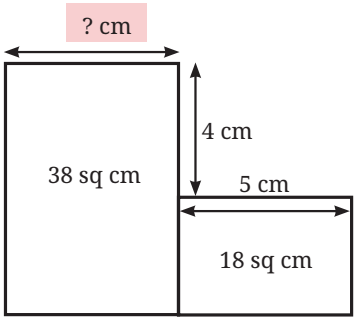
b.



c.



d.



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Figure it Out

1. Give the dimensions of a rectangle whose area is the sum of the areas of these two rectangles having measurements: 5 m × 10 m and 2 m × 7 m.
2. The area of a rectangular garden that is 50 m long is 1000 sq m. Find the width of the garden.
3. The floor of a room is 5 m long and 4 m wide. A square carpet whose sides are 3 m in length is laid on the floor. Find the area that is not carpeted.
4. Four flower beds having sides 2 m long and 1 m wide are dug at the four corners of a garden that is 15 m long and 12 m wide. How much area is now available for laying down a lawn?
5. Shape A has an area of 18 square units and Shape B has an area of 20 square units. Shape A has a longer perimeter than Shape B. Draw two such shapes satisfying the given conditions.
6. On a page in your book, draw a rectangular border that is 1 cm from the top and bottom and 1.5 cm from the left and right sides. What is the perimeter of the border?
7. Draw a rectangle of size 12 units × 8 units. Draw another rectangle inside it, without touching the outer rectangle that occupies exactly half the area.
8. A square piece of paper is folded in half. The square is then cut into two rectangles along the fold. Regardless of the size of the square, one of the following statements is always true. Which statement is true here?
   a. The area of each rectangle is larger than the area of the square.
   b. The perimeter of the square is greater than the perimeters of both the rectangles added together.
   c. The perimeters of both the rectangles added together is always $1\frac{1}{2}$ times the perimeter of the square.
   d. The area of the square is always three times as large as the areas of both rectangles added together.

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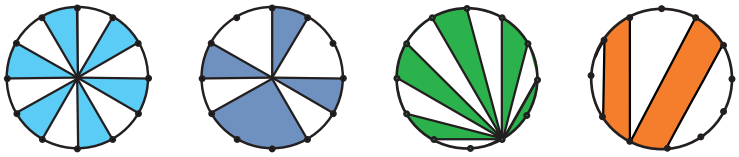
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SUMMARY

- The perimeter of a polygon is the sum of the lengths of all its sides.
  a. The perimeter of a rectangle is twice the sum of its length and width.
  b. The perimeter of a square is four times the length of any one of its sides.
- The area of a closed figure is the measure of the region enclosed by the figure.
- Area is generally measured in square units.
- The area of a rectangle is its length times its width. The area of a square is the length of any one of its sides multiplied by itself.
- Two closed figures can have the same area with different perimeters, or the same perimeter with different areas.
- Areas of regions can be estimated (or even determined exactly) by breaking up such regions into unit squares, or into more general-shaped rectangles and triangles whose areas can be calculated.

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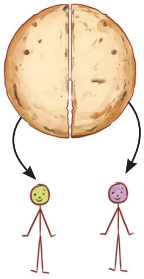


7 FRACTIONS

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Recall that when some whole number of things are shared equally among some number of people, fractions tell us how much each share is.

Shabnam: Do you remember, if one *roti* is divided equally between two children, how much *roti* will each child get?

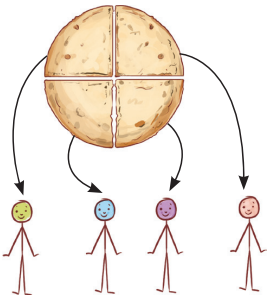


Mukta: Each child will get half a *roti*.

Shabnam: The fraction 'one half' is written as $\frac{1}{2}$. We also sometimes read this as 'one upon two.'

Mukta: If one *roti* is equally shared among 4 children, how much *roti* will one child get?

Shabnam: Each child's share is $\frac{1}{4}$ *roti*.



Mukta: And which is more $\frac{1}{2}$ *roti* or $\frac{1}{4}$ *roti*?

Shabnam: When 2 children share 1 *roti* equally, each child gets $\frac{1}{2}$ *roti*. When 4 children share 1 *roti* equally, each child gets $\frac{1}{4}$ *roti*. Since, in the second group more children share the

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same one *roti*, each child gets a smaller share. So, $\frac{1}{2}$ *roti* is more than $\frac{1}{4}$ *roti*.

$\frac{1}{2} > \frac{1}{4}$

7.1 Fractional Units and Equal Shares

Beni: Which fraction is greater — $\frac{1}{5}$ or $\frac{1}{9}$?

Arvin: 9 is bigger than 5. So I would guess that $\frac{1}{9}$ is greater than $\frac{1}{5}$. Am I right?

Beni: No! That is a common mistake. Think of these fractions as shares.

Arvin: If one *roti* is shared among 5 children, each one gets a share of $\frac{1}{5}$ *roti*. If one *roti* is shared among 9 children, each one gets a share of $\frac{1}{9}$ *roti*?

Beni: Exactly! Now think again - which share is higher?

Arvin: If I share with more people, I will get less. So, $\frac{1}{9} < \frac{1}{5}$.

Beni: You got it!

Oh, so $\frac{1}{100}$ is bigger than $\frac{1}{200}$!

When one unit is divided into several equal parts, each part is called a **fractional unit**. These are all fractional units:

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots, \frac{1}{10}, \ldots, \frac{1}{50}, \ldots, \frac{1}{100}$, etc.

We also sometimes refer to fractional units as '**unit fractions**.'

Figure it Out

Fill in the blanks with fractions.

1. Three guavas together weigh 1 kg. If they are roughly of the same size, each guava will roughly weigh ____ kg.
2. A wholesale merchant packed 1 kg of rice in four packets of equal weight. The weight of each packet is ___ kg.
3. Four friends ordered 3 glasses of sugarcane juice and shared it equally among themselves. Each one drank ____ glass of sugarcane juice.

Math Talk

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4. The big fish weighs $\frac{1}{2}$ kg. The small one weighs $\frac{1}{4}$ kg. Together they weigh ____ kg.



Knowledge from the past!

Fractions have been used and named in India since ancient times. In the *Rig Veda*, the fraction $\frac{3}{4}$ is referred to as *tri-pada*. This has the same meaning as the words for $\frac{3}{4}$ in many Indian languages today, e.g., '*teen paav*' in colloquial Hindi and '*mukkaal*' in Tamil. Indeed, words for fractions used today in many Indian languages go back to ancient times.

Find out and discuss the words for fractions that are used in the different languages spoken in your home, city, or state. Ask your grandparents, parents, teachers, and classmates what words they use for different fractions, such as for one and a half, three quarters, one and a quarter, half, quarter, and two and a half, and write them here:

_____ _____ _____ _____ _____ _____

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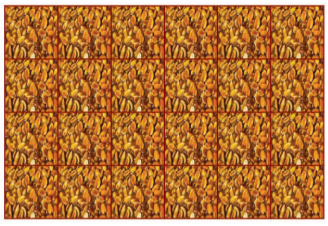
5. Arrange these fraction words in order of size from the smallest to the biggest in the empty box below:
   One and a half, three quarters, one and a quarter, half, quarter, two and a half.

Write your answer here.

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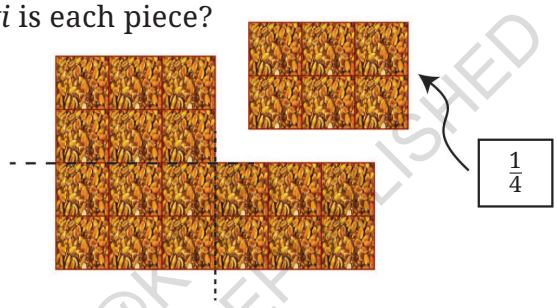
7.2 Fractional Units as Parts of a Whole

The picture shows a whole *chikki*.



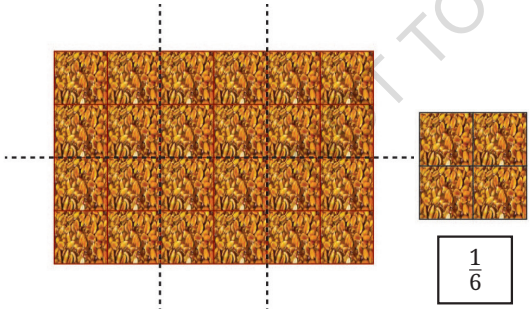
A whole *chikki*

A picture of the *chikki* broken into 2 pieces is shown below. How much of the original *chikki* is each piece?

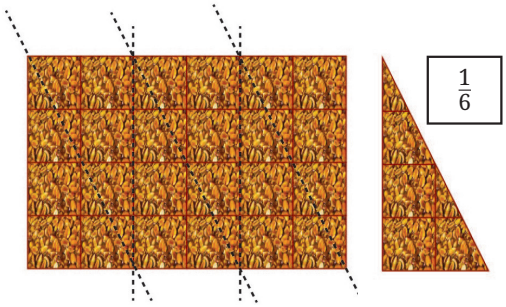


We can see that the bigger piece has 3 pieces of $\frac{1}{4}$ *chikki* in it. So, we can measure the bigger piece using the fractional unit $\frac{1}{4}$. We see that the bigger piece is $\frac{3}{4}$ *chikki*.

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A whole *chikki* cut into 6 equal pieces.



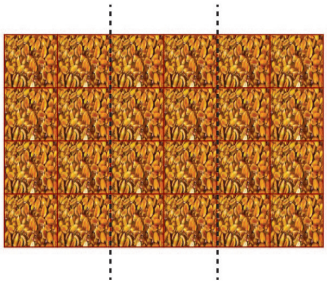
A whole *chikki* cut into 6 equal pieces in a different way.

By dividing the whole *chikki* into 6 equal parts in different ways, we get $\frac{1}{6}$ *chikki* pieces of different shapes. Are they of the same size?

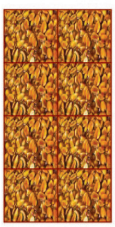
Math Talk

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What is the fractional unit of *chikki* shown below?



A whole *chikki*



$\frac{1}{3}$

We get this piece by breaking the *chikki* into 3 equal pieces. So this is $\frac{1}{3}$ *chikki*.

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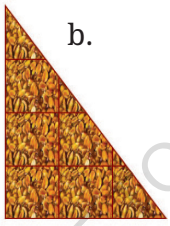
Figure it Out

The figures below show different fractional units of a whole *chikki*. How much of a whole *chikki* is each piece?

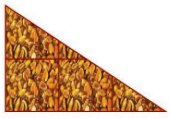
a.



b.



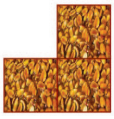
c.



d.



e.



f.



g.



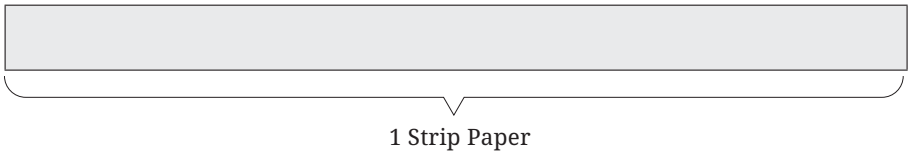
h.



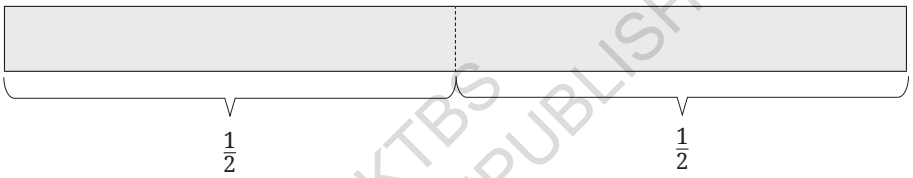
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7.3 Measuring Using Fractional Units

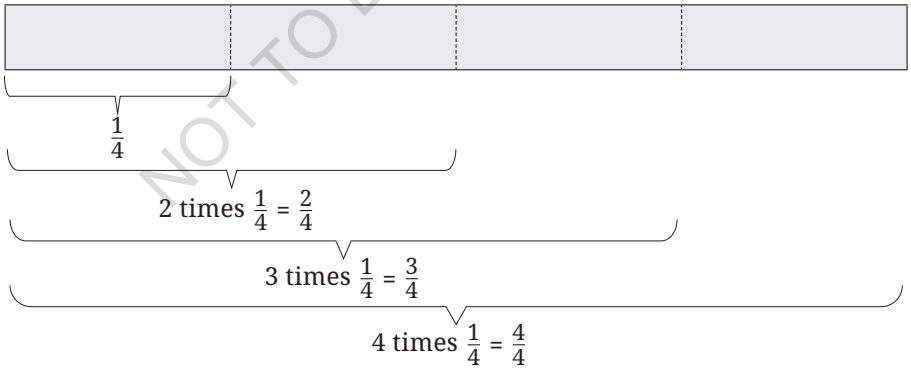
Take a strip of paper. We consider this paper strip to be one unit long.



Fold the strip into two equal parts and then open up the strip again. Taking the strip to be one unit in length, what are the lengths of the two new parts of the strip created by the crease?



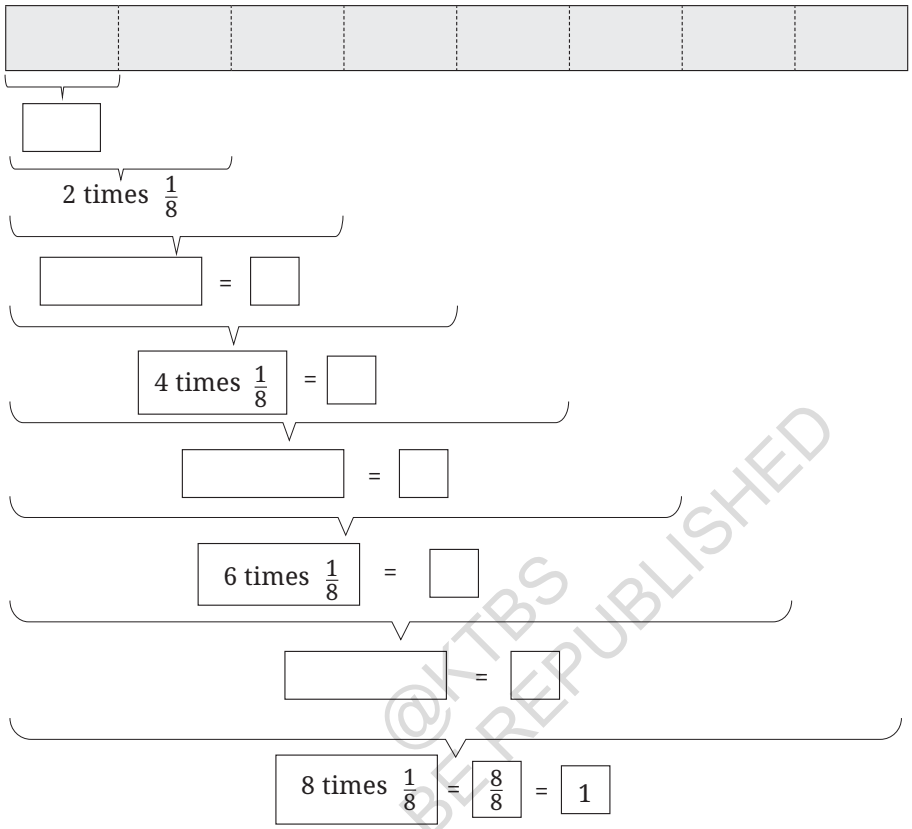
What will you get if you fold the previously-folded strip again into two equal parts? You will now get four equal parts.



Do it once more! Fill in the blank boxes.

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Fractional quantities can be measured using fractional units.

Let us look at another example,



Represents a full *roti* (whole)

				
$\frac{1}{2}$ = 1 times half	$\frac{1}{2} + \frac{1}{2}$ = 2 times half	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$ = 3 times half	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$ = 4 times half	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$ = 5 times half

We can describe how much the quantity is by collecting together the fractional units.

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Figure it Out

1. Continue this table of $\frac{1}{2}$ for 2 more steps.
2. Can you create a similar table for $\frac{1}{4}$?
3. Make $\frac{1}{3}$ using a paper strip. Can you use this to also make $\frac{1}{6}$?
4. Draw a picture and write an addition statement as above to show:
   a. 5 times $\frac{1}{4}$ of a *roti* b. 9 times $\frac{1}{4}$ of a *roti*
5. Match each fractional unit with the correct picture:

$\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{8}$ $\frac{1}{6}$



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Reading Fractions

We usually read the fraction $\frac{3}{4}$ as 'three quarters' or 'three upon four', but reading it as '3 times $\frac{1}{4}$' helps us to understand the size of the fraction because it clearly shows what the fractional unit is ($\frac{1}{4}$) and how many such fractional units (3) there are.

Recall what we call the top number and the bottom number of fractions. In the fraction $\frac{5}{6}$, 5 is the **numerator** and 6 is the **denominator.**

Teacher's Note

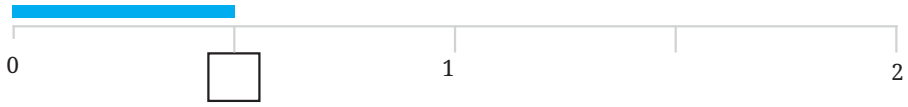
Give several opportunities to the children to explore the idea of fractional units with different shapes like circles, squares, rectangles, triangles, etc.

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7.4 Marking Fraction Lengths on the Number Line

We have marked lengths equal to 1, 2, 3, ... units on the number line. Now, let us try to mark lengths equal to fractions on the number line.

What is the length of the blue line? Write the fraction that gives the length of the blue line in the box.



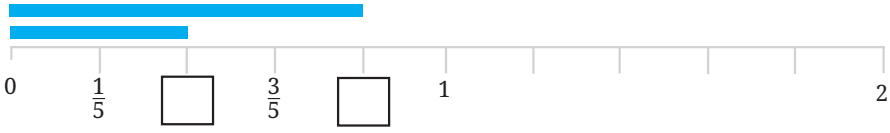
The distance between 0 and 1 is one unit long. It is divided into two equal parts. So, the length of each part is $\frac{1}{2}$ unit. So, this blue line is $\frac{1}{2}$ unit long.

Now, can you find the lengths of the various blue lines shown below? Fill in the boxes as well.

1. Here, the fractional unit is dividing a length of 1 unit into three equal parts. Write the fraction that gives the length of the blue line in the box or in your notebook.



2. Here, a unit is divided into 5 equal parts. Write the fraction that gives the length of the blue lines in the respective boxes or in your notebook.



3. Now, a unit is divided into 8 equal parts. Write the appropriate fractions in your notebook.

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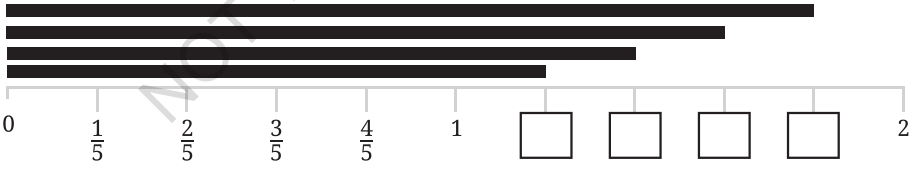
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Figure it Out

1. On a number line, draw lines of lengths $\frac{1}{10}$, $\frac{3}{10}$, and $\frac{4}{5}$.
2. Write five more fractions of your choice and mark them on the number line.
3. How many fractions lie between 0 and 1? Think, discuss with your classmates, and write your answer.
4. What is the length of the blue line and black line shown below? The distance between 0 and 1 is 1 unit long, and it is divided into two equal parts. The length of each part is $\frac{1}{2}$. So the blue line is $\frac{1}{2}$ units long. Write the fraction that gives the length of the black line in the box.



5. Write the fraction that gives the lengths of the black lines in the respective boxes.



Teacher's Note

Draw these lines on the board and ask the students to write the answers in their notebooks.

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7.5 Mixed Fractions

Fractions greater than one

You marked some fractions on the number line earlier. Did you notice that the lengths of all the blue lines were less than one and the lengths of all the black lines were more than 1?

Write down all the fractions you marked on the number line earlier.

Now, let us classify these in two groups:

Lengths less than 1 unit	Lengths more than 1 unit

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Did you notice something common between the fractions that are greater than 1?

In all the fractions that are less than 1 unit, the numerator is smaller than the denominator, while in the fractions that are more than 1 unit, the numerator is larger than the denominator.

We know that $\frac{3}{2}$, $\frac{5}{2}$ and $\frac{7}{2}$ are all greater than 1 unit. But can we see how many whole units they contain?

$\frac{3}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1 + \frac{1}{2}$

$\frac{5}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2 + \frac{1}{2}$

I know that $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$. If I add one more $\frac{1}{3}$, I will get more than 1 unit! So, $\frac{4}{3} > 1$.



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Figure it Out

1. How many whole units are there in $\frac{7}{2}$?

2. How many whole units are there in $\frac{4}{3}$ and in $\frac{7}{3}$?



Writing fractions greater than one as mixed numbers

We saw that: $\frac{3}{2} = 1 + \frac{1}{2}$.

We can write other fractions in a similar way. For example,

$$\frac{4}{3} = \underbrace{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}}_{3 \times \frac{1}{3} = 1} + \frac{1}{3} = 1 + \frac{1}{3}.$$

Figure it Out

1. Figure out the number of whole units in each of the following fractions:

   a. $\frac{8}{3}$ b. $\frac{11}{5}$ c. $\frac{9}{4}$

   We saw that

   $\frac{8}{3} = 2 + \frac{2}{3}$

   Fraction — Mixed number

   This number is thus also called 'two and two thirds'. We also write it as $2\frac{2}{3}$.

2. Can all fractions greater than 1 be written as such mixed numbers?

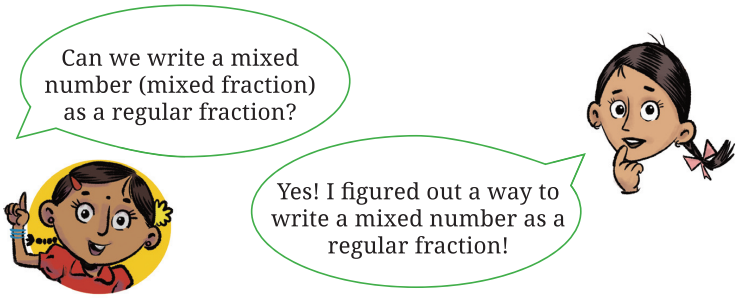
A **mixed number or mixed fraction** contains a whole number (called the whole part) and a fraction that is less than 1 (called the fractional part).

3. Write the following fractions as mixed fractions (e.g., $\frac{9}{2} = 4\frac{1}{2}$):

   a. $\frac{9}{2}$ b. $\frac{9}{5}$ c. $\frac{21}{19}$ d. $\frac{47}{9}$ e. $\frac{12}{11}$ f. $\frac{19}{6}$

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Jaya: When I have $3 + \frac{3}{4}$, this means $1 + 1 + 1 + \frac{3}{4}$. I know

$$1 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}.$$

So I get

$$(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}) + (\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}) + (\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}) + (\frac{1}{4} + \frac{1}{4} + \frac{1}{4}) = \frac{15}{4}.$$

Therefore, $(4 \times \frac{1}{4}) + (4 \times \frac{1}{4}) + (4 \times \frac{1}{4}) + (3 \times \frac{1}{4}) = \frac{15}{4}$.

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Figure it Out

Write the following mixed numbers as fractions:

a. $3\ \frac{1}{4}$ b. $7\ \frac{2}{3}$ c. $9\ \frac{4}{9}$

d. $3\ \frac{1}{6}$ e. $2\ \frac{3}{11}$ f. $3\ \frac{9}{10}$

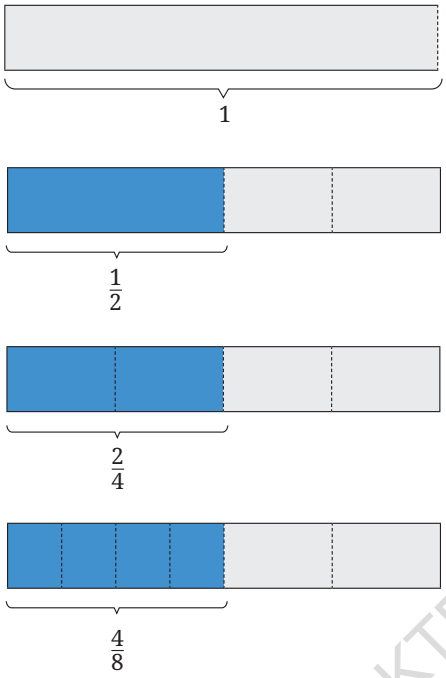


7.6 Equivalent Fractions

Using a fraction wall to find equal fractional lengths!

In the previous section, you used paper folding to represent various fractions using fractional units. Let us do some more activities with the same paper strips.

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What do you observe?

- Are the lengths $\frac{1}{2}$ and $\frac{2}{4}$ equal?
- Are the lengths $\frac{2}{4}$ and $\frac{4}{8}$ equal?

We can say that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

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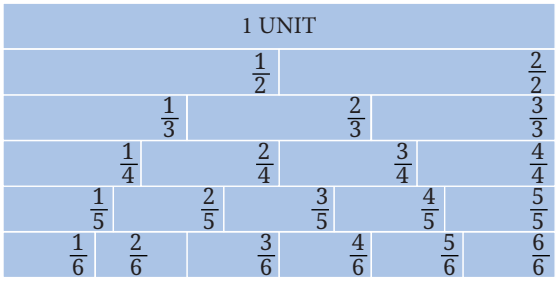
These are 'equivalent fractions' that denote the same length, but they are expressed in terms of different fractional units.

Now, check whether $\frac{1}{3}$ and $\frac{2}{6}$ are equivalent fractions or not, using paper strips.

Make your own fraction wall using such strips as given in the picture below!

Answer the following questions after looking at the fraction wall:

1. Are the lengths $\frac{1}{2}$ and $\frac{3}{6}$ equal?
2. Are $\frac{2}{3}$ and $\frac{4}{6}$ equivalent fractions? Why?
3. How many pieces of length $\frac{1}{6}$ will make a length of $\frac{1}{2}$?



4. How many pieces of length $\frac{1}{6}$ will make a length of $\frac{1}{3}$?

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We can extend this idea to make a fraction wall up to the fractional unit $\frac{1}{10}$. (This fraction wall is given at the end of the book.)

1 UNIT									
				$\frac{1}{2}$					$\frac{2}{2}$
			$\frac{1}{3}$			$\frac{2}{3}$			$\frac{3}{3}$
		$\frac{1}{4}$			$\frac{2}{4}$			$\frac{3}{4}$	$\frac{4}{4}$
	$\frac{1}{5}$			$\frac{2}{5}$			$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$
	$\frac{1}{6}$		$\frac{2}{6}$			$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$
$\frac{1}{7}$		$\frac{2}{7}$			$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$	$\frac{1}{7}$
$\frac{1}{8}$	$\frac{2}{8}$		$\frac{3}{8}$		$\frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$	$\frac{8}{8}$
$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$	$\frac{9}{9}$	
$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$

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Figure it Out

1. Are $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$ equivalent fractions? Why?

2. Write two equivalent fractions for $\frac{2}{6}$.

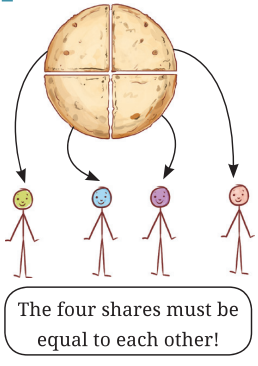
3. $\frac{4}{6}$ = $\frac{\quad}{\quad}$ = $\frac{\quad}{\quad}$ = $\frac{\quad}{\quad}$ = ... (Write as many as you can)

Understanding equivalent fractions using equal shares

One *roti* was shared equally by four children. What fraction of the whole did each child get?

The adjoining picture shows the division of a *roti* among four children.

Fraction of *roti* each child got is $\frac{1}{4}$.



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You can also express this event through division facts, addition facts, and multiplication facts.

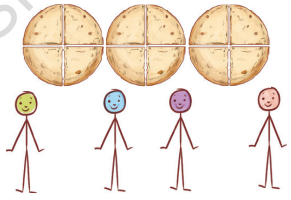
The division fact is $1 \div 4 = \frac{1}{4}$.

The addition fact is $1 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.

The multiplication fact is $1 = 4 \times \frac{1}{4}$.

Figure it Out

1. Three *rotis* are shared equally by four children. Show the division in the picture and write a fraction for how much each child gets. Also, write the corresponding division facts, addition facts, and, multiplication facts.

   Fraction of *roti* each child gets is _____.

   Division fact:

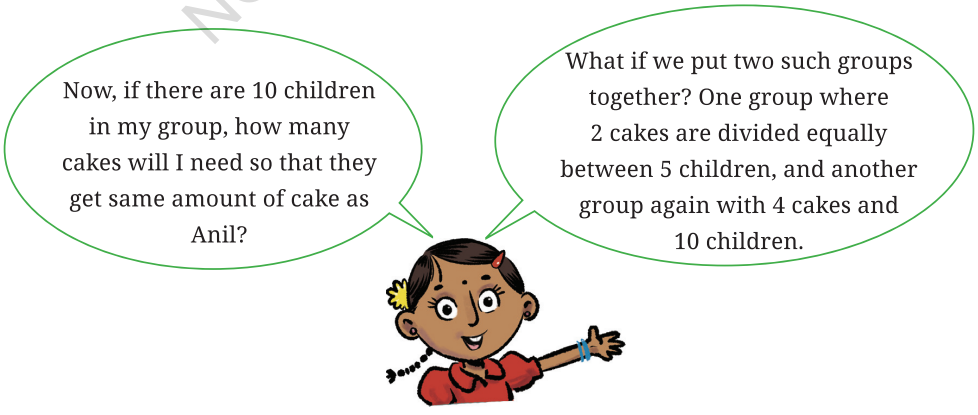
   Addition fact:

   Multiplication fact:

   Compare your picture and answers with your classmates!

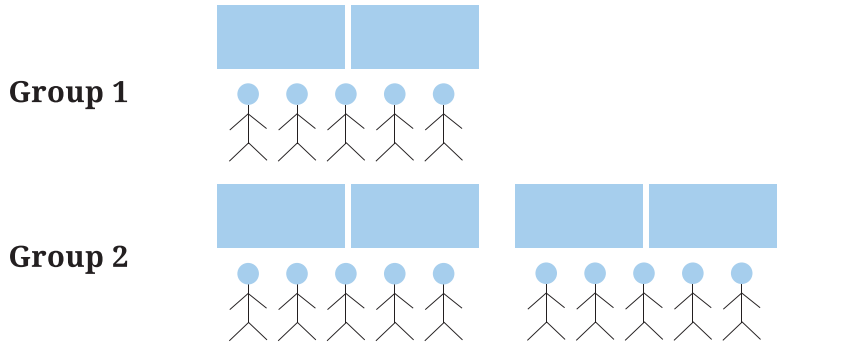
2. Draw a picture to show how much each child gets when 2 *rotis* are shared equally by 4 children. Also, write the corresponding division facts, addition facts, and multiplication facts.

3. Anil was in a group where 2 cakes were divided equally among 5 children. How much cake would Anil get?

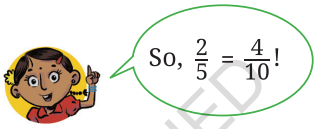


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So, the share of each child is the same in both these situations!

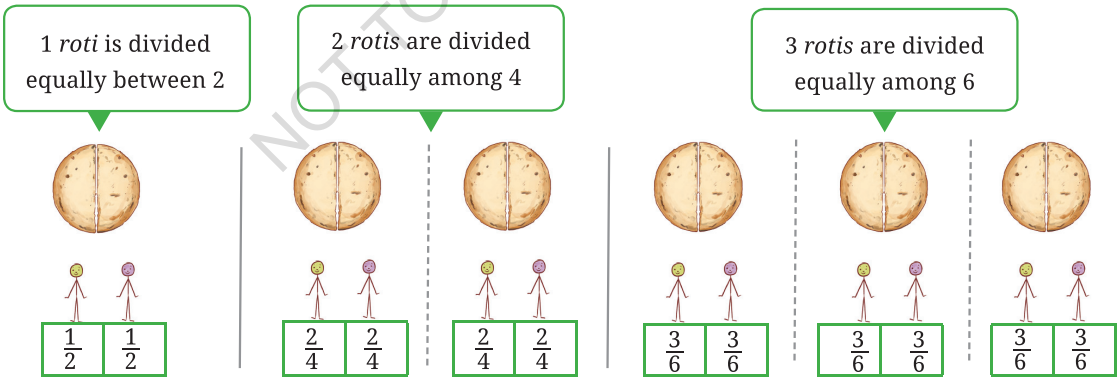


Let us examine the shares of each child in the following situations.

- 1 *roti* is divided equally between 2 children.
- 2 *rotis* are divided equally among 4 children.
- 3 *rotis* are divided equally among 6 children.

Let us draw and share!

Did you notice that in each situation the share of every child is the same? So, we can say that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$.



Fractions where the shares are equal are called 'equivalent fractions'.

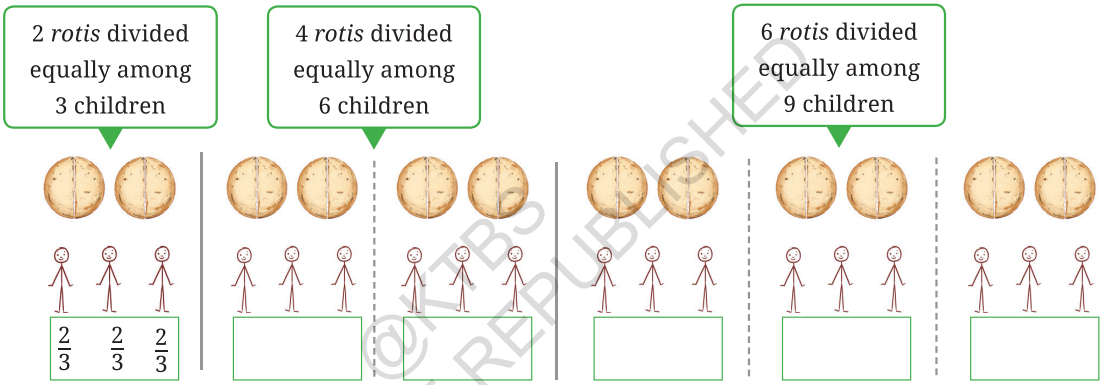
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So, $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ are all **equivalent fractions**.

Find some more fractions equivalent to $\frac{1}{2}$. Write them in the boxes here: ☐ ☐ ☐ ☐

Equally divide the *rotis* in the situations shown below and write down the share of each child. Are the shares in each of these cases the same? Why?



$\frac{2}{3}$ is also called the simplest form of $\frac{4}{6}$. It is also the simplest form of $\frac{6}{9}$ as well.

Do you notice anything about the relationship between the numerator and denominator in each of these fractions?



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Figure it Out

Find the missing numbers:

Math Talk

a. 5 glasses of juice shared equally among 4 friends is the same as ____ glasses of juice shared equally among 8 friends.

So, $\frac{5}{4} = \frac{\square}{8}$.

b. 4 kg of potatoes divided equally in 3 bags is the same as 12 kgs of potatoes divided equally in ___ bags.

So, $\frac{4}{3} = \frac{12}{\square}$

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c. 7 *rotis* divided among 5 children is the same as ____ *rotis* divided among _____ children.

So, $\frac{7}{5} = \frac{\square}{\square}$.

In which group will each child get more *chikki*?

1 *chikki* divided between 2 children or 5 *chikkis* divided among 8 children.

Mukta: So, we must compare $\frac{1}{2}$ and $\frac{5}{8}$. Which is more?

Shabnam: Well, we have seen that $\frac{1}{2} = \frac{4}{8}$; and clearly $\frac{4}{8} < \frac{5}{8}$. So, the children for whom 5 *chikkis* is divided equally among 8 will get more than those children for whom 1 *chikki* is divided equally among 2. The children of the second group will get more *chikki* each.

What about the following groups? In which group will each child get more?

1 *chikki* divided between 2 children or 4 *chikkis* divided among 7 children.

Shabnam: The children of which group will get more *chikki* this time?

Mukta: We must compare $\frac{1}{7}$ and $\frac{4}{7}$.

Now

$\frac{1 \times 4}{2 \times 4} = \frac{4}{8}$ so, $\frac{1}{2} = \frac{4}{8}$.

Shabnam: But why did you multiply the numerator and denominator by 4 again?

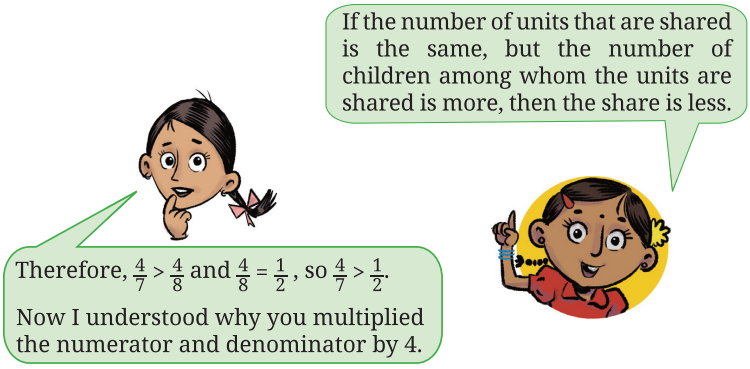
Mukta: You will see!

When 4 *chikkis* are divided equally among 7 children, each one will get $\frac{4}{7}$ *chikki*. When 4 *chikkis* are divided equally among 8 children, each one will get $\frac{4}{8}$ *chikki*. So $\frac{4}{7} > \frac{4}{8}$.



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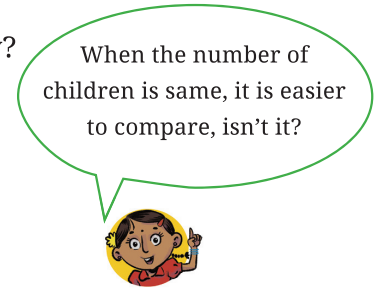
Suppose the number of children is kept the same, but the number of units that are being shared is increased? What can you say about each child's share now? Why? Discuss how your reasoning explains $\frac{1}{5} < \frac{2}{5}$, $\frac{3}{7} < \frac{4}{7}$, and $\frac{1}{2} < \frac{5}{8}$.

Now, decide in which of the two groups will each child get a larger share:

1. **Group 1 :** 3 glasses of sugarcane juice divided equally among 4 children.
   Group 2: 7 glasses of sugarcane juice divided equally among 10 children.
2. **Group 1 :** 4 glasses of sugarcane juice divided equally among 7 children.
   Group 2: 5 glasses of sugarcane juice divided equally among 7 children.

Math Talk

Which groups were easier to compare? Why?



Shabnam: To compare the first two groups, we have to find fractions equivalent to the fractions $\frac{3}{4}$ and $\frac{7}{10}$.

Mukta: How about $\frac{6}{8} = \frac{3}{4}$ and $\frac{21}{30} = \frac{7}{10}$?

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Shabnam: There is a condition. The fractional unit used for the two fractions have to be the same! Like $\frac{2}{6}$ and $\frac{3}{6}$ both use the same fractional unit $\frac{1}{6}$ (i.e., the denominators are the same). But $\frac{6}{8}$ and $\frac{21}{30}$ do not use the same fractional units (they have different denominators).

Mukta: Okay, so let us start making equivalent fractions then:

$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20}$ ... But when do I stop?

Shabnam: Got it! How about we go on till $4 \times 10 = 40$.

Mukta: You mean the product of the two denominators? Sounds good!

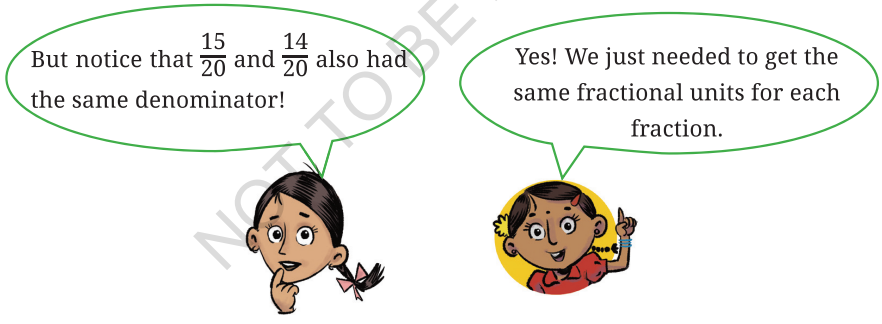
We have $\frac{3}{4}$ and $\frac{7}{10}$. The product of the two denominators (4 and 10) is 40.

$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24} = \ldots = \frac{27}{36} = \frac{30}{40}$.

$\frac{7}{10} = \frac{14}{20} = \frac{21}{30} = \frac{28}{40}$.

Go till we reach the denominator 40.

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Shabnam: So, fractions equivalent to $\frac{3}{4}$ and $\frac{7}{10}$ with the same fractional unit (same denominators) are $\frac{30}{40}$ and $\frac{28}{40}$, or $\frac{15}{20}$ and $\frac{14}{20}$.

Since clearly $\frac{30}{40} > \frac{28}{40}$, we conclude that $\frac{3}{4} > \frac{7}{10}$.

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Find equivalent fractions for the given pairs of fractions such that the fractional units are the same.

a. $\frac{7}{2}$ and $\frac{3}{5}$ b. $\frac{8}{3}$ and $\frac{5}{6}$ c. $\frac{3}{4}$ and $\frac{3}{5}$ d. $\frac{6}{7}$ and $\frac{8}{5}$

e. $\frac{9}{4}$ and $\frac{5}{2}$ f. $\frac{1}{10}$ and $\frac{2}{9}$ g. $\frac{8}{3}$ and $\frac{11}{4}$ h. $\frac{13}{6}$ and $\frac{1}{9}$

Expressing a fraction in lowest terms (or in its simplest form)

In any fraction, if its numerator and denominator have no common factor except 1, then the fraction is said to be in **lowest terms** or in its **simplest form**. In other words, a fraction is said to be in lowest terms if its numerator and denominator are as small as possible.

Any fraction can be expressed in lowest terms by finding an equivalent fraction whose numerator and denominator are as small as possible.

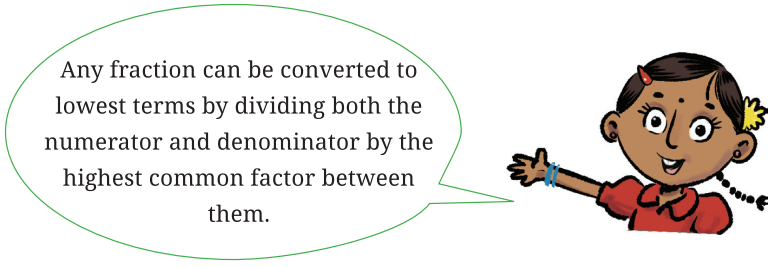
Let's see how to express fractions in lowest terms.

Example: Is the fraction $\frac{16}{20}$ in lowest terms? No, 4 is a common factor of 16 and 20. Let us reduce $\frac{16}{20}$ to lowest terms.

We know that both 16 (numerator) and 20 (denominator) are divisible by 4.

So, $\frac{16 \div 4}{20 \div 4} = \frac{4}{5}$.

Now, there is no common factor between 4 and 5. Hence, $\frac{16}{20}$ expressed in lowest terms is $\frac{4}{5}$. So, $\frac{4}{5}$ is called the simplest form of $\frac{16}{20}$, since 4 and 5 have no common factor other than 1.



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Expressing a fraction in lowest terms can also be done in steps.

Suppose we want to express $\frac{36}{60}$ in lowest terms. First, we notice that both the numerator and denominator are even. So, we divide both by 2, and see that $\frac{36}{60} = \frac{18}{30}$.

Both the numerator and denominator are even again, so we can divide them each by 2 again; we get $\frac{18}{30} = \frac{9}{15}$.

We now notice that 9 and 15 are both multiples of 3, so we divide both by 3 to get $\frac{9}{15} = \frac{3}{5}$.

Now, 3 and 5 have no common factor other than 1, so, $\frac{36}{60}$ in lowest terms is $\frac{3}{5}$.

Alternatively, we could have noticed that in $\frac{36}{60}$, both the numerator and denominator are multiples of 12 : we see that $36 = 3 \times 12$ and $60 = 5 \times 12$. Therefore, we could have concluded that $\frac{36}{60} = \frac{3}{5}$ straight away.

Either method works and will give the same answer! But sometimes it can be easier to go in steps.

Figure it Out

Express the following fractions in lowest terms:

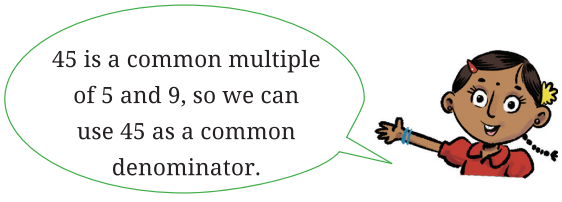
a. $\frac{17}{51}$ b. $\frac{64}{144}$ e. $\frac{126}{147}$ d. $\frac{525}{112}$

7.7 Comparing Fractions

Which is greater, $\frac{4}{5}$ or $\frac{7}{9}$? It can be difficult to compare two such fractions directly. However, we know how to find fractions equivalent to two fractions with the same denominator. Let us see how we can use it:

$\frac{4}{5} = \frac{4\times9}{5\times9} = \frac{36}{45}$

$\frac{7}{9} = \frac{7\times5}{8\times5} = \frac{35}{45}$.



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Clearly, $\frac{36}{45} > \frac{35}{45}$

So, $\frac{4}{5} > \frac{7}{9}$!

Let us try this for another pair: $\frac{7}{9}$ and $\frac{17}{21}$.

63 is a common multiple of 9 and 21. We can then write:

$\frac{7}{9} = \frac{7\times7}{9\times7} = \frac{49}{63}$, $\frac{17}{21} = \frac{17\times3}{21\times3} = \frac{51}{63}$.

Clearly, $\frac{49}{63} < \frac{51}{63}$. So, $\frac{7}{9} < \frac{17}{21}$!

Let's Summarise!

Steps to compare the sizes of two or more given fractions:

Step 1: Change the given fractions to equivalent fractions so that they all are expressed with the same denominator or same fractional unit.

Step 2: Now, compare the equivalent fractions by simply comparing the numerators, i.e., the number of fractional units each has.

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Figure it Out

1. Compare the following fractions and justify your answers:

   a. $\frac{8}{3}, \frac{5}{2}$ b. $\frac{4}{9}, \frac{3}{7}$ c. $\frac{7}{10}, \frac{9}{14}$

   d. $\frac{12}{5}, \frac{8}{5}$ e. $\frac{9}{4}, \frac{5}{2}$

2. Write the following fractions in ascending order.

   a. $\frac{7}{10}, \frac{11}{15}, \frac{2}{5}$ b. $\frac{19}{24}, \frac{5}{6}, \frac{7}{12}$

3. Write the following fractions in descending order.

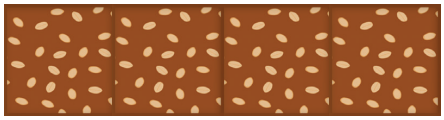
   a. $\frac{25}{16}, \frac{7}{8}, \frac{13}{4}, \frac{17}{32}$ b. $\frac{3}{4}, \frac{12}{5}, \frac{7}{12}, \frac{5}{4}$

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7.8 Addition and Subtraction of Fractions

Meena's father made some *chikki*. Meena ate $\frac{1}{2}$ of it and her younger brother ate $\frac{1}{4}$ of it. How much of the total *chikki* did Meena and her brother eat together?





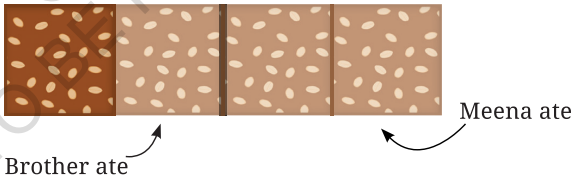
We can arrive at the answer by visualising it. Let us take a piece of *chikki* and divide it into two halves first like this.

Meena ate $\frac{1}{2}$ of it as shown in the picture.

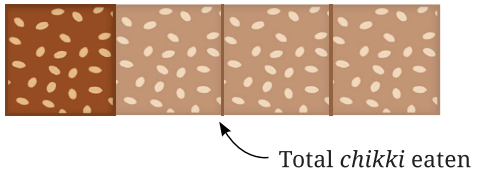


Let us now divide the remaining half into two further halves as shown. Each of these pieces is $\frac{1}{4}$ of the whole *chikki*.

Meena's brother ate $\frac{1}{4}$ of the whole *chikki*, as is shown in the picture.



The total *chikki* eaten is $\frac{1}{2}$ (by Meena) and $\frac{1}{4}$ (by her brother)



The total *chikki* eaten

$= \frac{1}{2} + \frac{1}{4}$

$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

$= 3 \times \frac{1}{4} = \frac{3}{4}$.

How much of the total *chikki* is remaining?

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Adding fractions with the same fractional unit or denominator

Example: Find the sum of $\frac{2}{5}$ and $\frac{1}{5}$.

Let us represent both using the rectangular strips. In both fractions, the fractional unit is the same $\frac{1}{5}$, so, each strip will be divided into 5 equal parts.

So $\frac{2}{5}$ will be represented as—



And $\frac{1}{5}$ will be represented as—



Adding the two given fractions is the same as finding out the total number of shaded parts, each of which represent the same fractional unit $\frac{1}{5}$.

In this case, the total number of shaded parts is 3. Since, each shaded part represents the fractional unit $\frac{1}{5}$, we see that the 3 shaded parts together represent the fraction $\frac{3}{5}$.

Therefore, $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$?



Example: Find the sum of $\frac{4}{7}$ and $\frac{6}{7}$.

Let us represent both again using the rectangular strip model. Here in both fractions, the fractional unit is the same, i.e., $\frac{1}{7}$, so each strip will be divided into 7 equal parts.

Then $\frac{4}{7}$ will be represented as—



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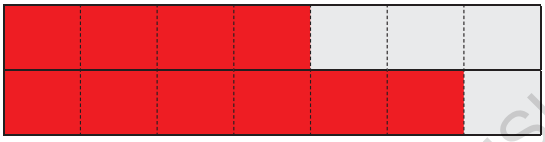
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and $\frac{6}{7}$ will be represented as—



In this case, the total number of shaded parts is 10, and each shaded part represents the fractional unit $\frac{1}{7}$, so, the 10 shaded parts together represent the fraction $\frac{10}{7}$ as seen here.

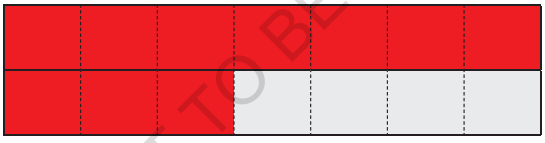
While adding fractions with the same fractional unit, just add the number of fractional units from each fraction.



Therefore, $\frac{4}{7} + \frac{6}{7} = \frac{10}{7}$

$= 1 + \frac{3}{7}$

$= 1\frac{3}{7}$.



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Try adding $\frac{4}{7} + \frac{6}{7}$ using a number line. Do you get the same answer?

Adding fractions with different fractional units or denominators

Example: Find the sum of $\frac{1}{4}$ and $\frac{1}{3}$.

To add fractions with different fractional units, first convert the fractions into equivalent fractions with the same denominator or

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fractional unit. In this case, the common denominator can be made $3 \times 4 = 12$, i.e., we can find equivalent fractions with fractional unit $\frac{1}{12}$.

Let us write the equivalent fraction for each given fraction.

$\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}$, $\frac{1}{3} = \frac{1 \times 4}{3 \times 4} = \frac{4}{12}$.

Now, $\frac{3}{12}$ and $\frac{4}{12}$ have the same fractional unit, i.e., $\frac{1}{12}$.

Therefore, $\frac{1}{4} + \frac{1}{3} = \frac{3}{12} + \frac{4}{12} = \frac{7}{12}$.

This method of addition, which works for adding any number of fractions, was first explicitly described in general by Brahmagupta in the year 628 CE! We will describe the history of the development of fractions in more detail later in the chapter. For now, we simply summarise the steps in Brahmagupta's method for addition of fractions.

Brahmagupta's method for adding fractions

1. Find equivalent fractions so that the fractional unit is common for all fractions. This can be done by finding a common multiple of the denominators (e.g., the product of the denominators, or the smallest common multiple of the denominators).
2. Add these equivalent fractions with the same fractional units. This can be done by adding the numerators and keeping the same denominator.
3. Express the result in lowest terms if needed.

Let us carry out another example of Brahmagupta's method.

Example: Find the sum of $\frac{2}{3}$ and $\frac{1}{5}$.

The denominators of the given fractions are 3 and 5. The lowest common multiple of 3 and 5 is 15. Then we see that

$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$, $\frac{1}{5} = \frac{1 \times 3}{5 \times 3} = \frac{3}{15}$.

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Therefore, $\frac{2}{3} + \frac{1}{5} = \frac{10}{15} + \frac{3}{15} = \frac{13}{15}$.

Example: Find the sum of $\frac{1}{6}$ and $\frac{1}{3}$.

The smallest common multiple of 6 and 3 is 6.

$\frac{1}{6}$ will remain $\frac{1}{6}$.

$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$

Therefore, $\frac{1}{6} + \frac{1}{3} = \frac{1}{6} + \frac{2}{6} = \frac{3}{6}$.

The fraction $\frac{3}{6}$ can now be re-expressed in lowest terms, if desired. This can be done by dividing both the numerator and denominator by 3 (the biggest common factor of 3 and 6):

$\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$.

Therefore, $\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$.

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Figure it Out

1. Add the following fractions using Brahmagupta's method:

   a. $\frac{2}{7} + \frac{5}{7} + \frac{6}{7}$ b. $\frac{3}{4} + \frac{1}{3}$ c. $\frac{2}{3} + \frac{5}{6}$ d. $\frac{2}{3} + \frac{2}{7}$ e. $\frac{3}{4} + \frac{1}{3} + \frac{1}{5}$

   f. $\frac{2}{3} + \frac{4}{5}$ g. $\frac{4}{5} + \frac{2}{3}$ h. $\frac{3}{5} + \frac{5}{8}$ i. $\frac{9}{2} + \frac{5}{4}$ j. $\frac{8}{3} + \frac{2}{7}$

   k. $\frac{3}{4} + \frac{1}{3} + \frac{1}{5}$ l. $\frac{2}{3} + \frac{4}{5} + \frac{3}{7}$ m. $\frac{9}{2} + \frac{5}{4} + \frac{7}{6}$

2. Rahim mixes $\frac{2}{3}$ litres of yellow paint with $\frac{3}{4}$ litres of blue paint to make green paint. What is the volume of green paint he has made?
3. Geeta bought $\frac{2}{5}$ meter of lace and Shamim bought $\frac{3}{4}$ meter of the same lace to put a complete border on a table cloth whose perimeter is 1 meter long. Find the total length of the lace they both have bought. Will the lace be sufficient to cover the whole border?

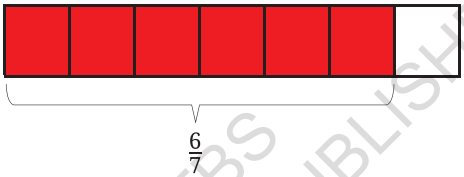
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Subtraction of fractions with the same fractional unit or denominator

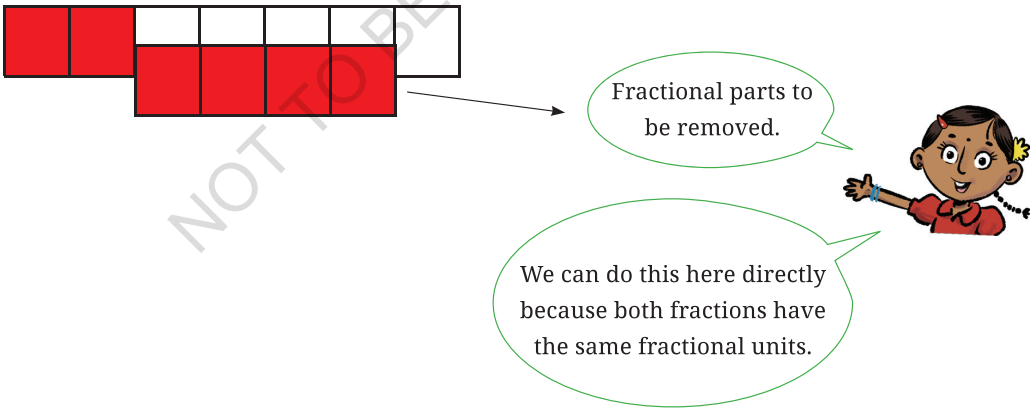
Brahmagupta's method also applies when subtracting fractions!

Let us start with the problem of subtracting $\frac{4}{7}$ from $\frac{6}{7}$, i.e., what is $\frac{6}{7} - \frac{4}{7}$?

To solve this problem, we can again use the rectangular strips. In both fractions, the fractional unit is the same, i.e., $\frac{1}{7}$. Let us first represent the bigger fraction using a rectangular strip model as shown:



Each shaded part represents $\frac{1}{7}$. Now, we need to subtract $\frac{4}{7}$. To do this let us remove 4 of the shaded parts:



So, we are left with 2 shaded parts, i.e., $\frac{6}{7} - \frac{4}{7} = \frac{2}{7}$.

Try doing this same exercise using the number line.

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Figure it Out

1. $\frac{5}{8} - \frac{3}{8}$ 2. $\frac{7}{9} - \frac{5}{9}$ 3. $\frac{10}{27} - \frac{1}{27}$

Subtraction of fractions with different fractional units or denominators

Example: What is $\frac{3}{4} - \frac{2}{3}$?

As we already know the procedure for subtraction of fractions with the same fractional units, let us convert each of the given fractions into equivalent fractions with the same fractional units.

$\frac{3}{4} = \frac{(3\times3)}{(4\times3)} = \frac{9}{12}$

Yes! By doing this we can easily subtract the two fractions.

Think! Why did we choose to multiply both the numerator and denominator by 3?

and similarly,
$\frac{2}{3} = \frac{(2\times4)}{(3\times4)} = \frac{8}{12}$.

Again! Why did we choose to multiply both the numerator and denominator here by 4?

Therefore, $\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$.

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Brahmagupta's method for subtracting two fractions —

1. Convert the given fractions into equivalent fractions with the same fractional unit, i.e., the same denominator.
2. Carry out the subtraction of fractions having the same fractional units. This can be done by subtracting the numerators and keeping the same denominator.
3. Simplify the result into lowest terms if needed.

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Figure it Out

1. Carry out the following subtractions using Brahmagupta's method:
   a. $\frac{8}{15} - \frac{3}{15}$ b. $\frac{2}{5} - \frac{4}{15}$ c. $\frac{5}{6} - \frac{4}{9}$ d. $\frac{2}{3} - \frac{1}{2}$

2. Subtract as indicated:
   a. $\frac{13}{4}$ from $\frac{10}{3}$ b. $\frac{18}{5}$ from $\frac{23}{3}$ c. $\frac{29}{7}$ from $\frac{45}{7}$

3. Solve the following problems:
   a. Jaya's school is $\frac{7}{10}$ km from her home. She takes an auto for $\frac{1}{2}$ km from her home daily, and then walks the remaining distance to reach her school. How much does she walk daily to reach the school?
   b. Jeevika takes $\frac{10}{3}$ minutes to take a complete round of the park and her friend Namit takes $\frac{13}{4}$ minutes to do the same. Who takes less time and by how much?

7.9 A Pinch of History

Do you know what a fraction was called in ancient India? It was called *bhinna* in Sanskrit, which means 'broken'. It was also called *bhaga* or *ansha* meaning 'part' or 'piece'.

The way we write fractions today, globally, originated in India. In ancient Indian mathematical texts, such as the *Bakshali manuscript* (from around the year 300 CE), when they wanted to write $\frac{1}{2}$, they wrote it as $\frac{1}{2}$ which is indeed very similar to the way we write it today! This method of writing and working with fractions continued to be used in India for the next several centuries, including by Aryabhata (499 CE), Brahmagupta (628 CE), Sridharacharya (c. 750 CE), and Mahaviracharya (c. 850 CE), among others. The line segment between the numerator and denominator in '$\frac{1}{2}$' and in other

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fractions was later introduced by the Moroccan mathematician Al-Hassar (in the 12th century). Over the next few centuries the notation then spread to Europe and around the world.

Fractions had also been used in other cultures such as the ancient Egyptian and Babylonian civilisations, but they primarily used only fractional units, that is, fractions with a 1 in the numerator. More general fractions were expressed as sums of fractional units, now called 'Egyptian fractions'. Writing numbers as the sum of fractional units, e.g., $\frac{19}{24} = \frac{1}{2} + \frac{1}{6} + \frac{1}{8}$, can be quite an art and leads to beautiful puzzles. We will consider one such puzzle below.

General fractions (where the numerator is not necessarily 1) were first introduced in India, along with their rules of arithmetic operations like addition, subtraction, multiplication, and even division of fractions. The ancient Indian treatises called the 'Sulba-sutras' shows that even during Vedic times, Indians had discovered the rules for operations with fractions. General rules and procedures for working with and computing with fractions were first codified formally and in a modern form by Brahmagupta.

Brahmagupta's methods for working with and computing with fractions are still what we use today. For example, Brahmagupta described how to add and subtract fractions as follows:

"By the multiplication of the numerator and the denominator of each of the fractions by the other denominators, the fractions are reduced to a common denominator. Then, in case of addition, the numerators (obtained after the above reduction) are added. In case of subtraction, their difference is taken." (Brahmagupta, Brahmasphuṭasiddhānta, Verse 12.2, 628 CE)

The Indian concepts and methods involving fractions were transmitted to Europe via the Arabs over the next few centuries and they came into general use in Europe in around the 17th century and then spread worldwide.

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Puzzle!

It is easy to add up fractional units to obtain the sum 1, if one uses the same fractional unit, for example,

$\frac{1}{2} + \frac{1}{2} = 1$, $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$, $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$, etc.

However, can you think of a way to add fractional units that are all different to get 1?

It is not possible to add two different fractional units to get 1. The reason is that $\frac{1}{2}$ is the largest fractional unit, and $\frac{1}{2} + \frac{1}{2} = 1$.

To get different fractional units, we would have to replace at least one of the $\frac{1}{2}$'s with some smaller fractional unit - but then the sum would be less than 1! Therefore, it is not possible for two different fractional units to add up to 1.

We can try to look instead for a way to write 1 as the sum of three different fractional units.

Try This

1. Can you find three different fractional units that add up to 1?

   It turns out there is only one solution to this problem (up to changing the order of the 3 fractions)! Can you find it? Try to find it before reading further.

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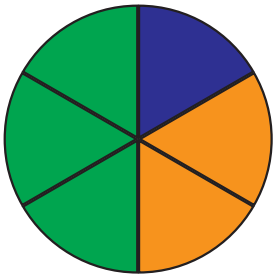
Here is a systematic way to find the solution. We know that $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$. To get the fractional units to be different, we will have to increase at least one of the $\frac{1}{3}$'s, and decrease at least one of the other $\frac{1}{3}$'s to compensate for that increase. The only way to increase $\frac{1}{3}$ to another fractional unit is to replace it by $\frac{1}{2}$. So $\frac{1}{2}$ must be one of the fractional units.

Now $\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$. To get the fractional units to be different, we will have to increase one of the $\frac{1}{4}$'s and decrease the other $\frac{1}{4}$ to compensate for that increase. Now the only way to increase $\frac{1}{4}$ to

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another fractional unit, that is different from $\frac{1}{2}$, is to replace it by $\frac{1}{3}$. So two of the fractions must be $\frac{1}{2}$ and $\frac{1}{3}$! What must be third fraction then, so that the three fractions add up to 1?

This explains why there is only one solution to the above problem.



$$\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$$

What if we look for four different fractional units that add up to 1?

Try This

2. Can you find four different fractional units that add up to 1?

   It turns out that this problem has six solutions! Can you find at least one of them? Can you find them all? You can try using similar reasoning as in the cases of two and three fractional units—or find your own method!

Once you find one solution, try to divide a circle into parts like in the figure above to visualise it!

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SUMMARY

- **Fraction as equal share:** When a whole number of units is divided into equal parts and shared equally, a **fraction** results.
- **Fractional Units:** When one whole basic unit is divided into equal parts, then each part is called a **fractional unit.**
- **Reading Fractions:** In a fraction such as $\frac{5}{6}$, 5 is called the **numerator** and 6 is called the **denominator.**
- **Mixed fractions** contain a whole number part and a fractional part.
- **Number line:** Fractions can be shown on a number line. Every fraction has a point associated with it on the number line.
- **Equivalent Fractions:** When two or more fractions represent the same share or number, they are called **equivalent fractions**.
- **Lowest terms:** A fraction whose numerator and denominator have no common factor other than 1 is said to be in **lowest terms** or in its **simplest form.**
- **Brahmagupta's method for adding fractions:** When adding fractions, convert them into equivalent fractions with the same fractional unit (i.e., the same denominator), and then add the number of fractional units in each fraction to obtain the sum. This is accomplished by adding the numerators while keeping the same denominator.
- **Brahmagupta's method for subtracting fractions:** When subtracting fractions, convert them into equivalent fractions with the same fractional unit (i.e., the same denominator), and then subtract the number of fractional units. This is accomplished by subtracting the numerators while keeping the same denominator.

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8 PLAYING WITH CONSTRUCTIONS

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8.1 Artwork

Observe the following figures and try drawing them freehand.

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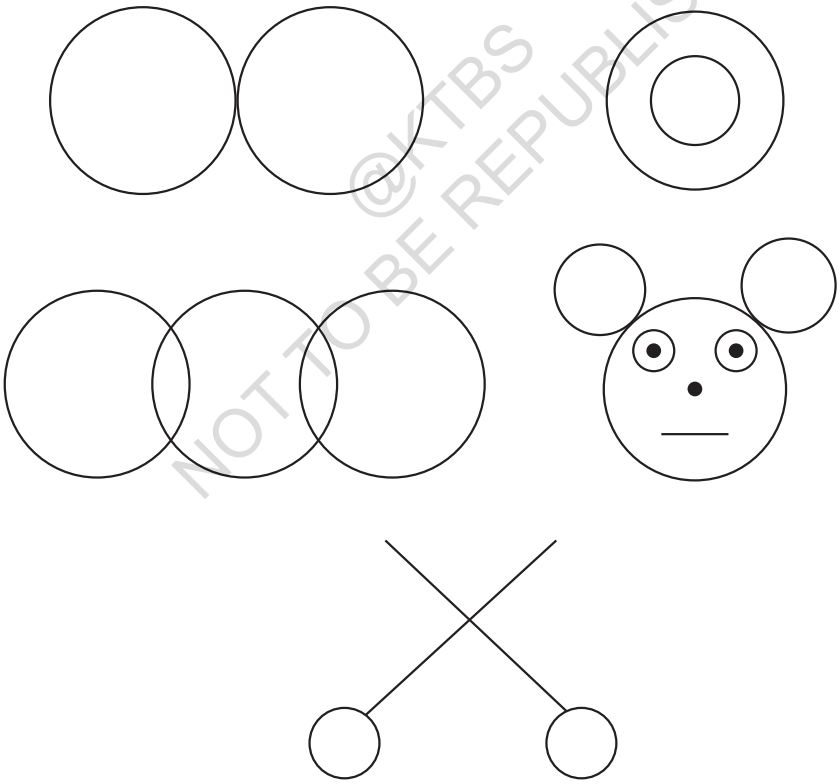
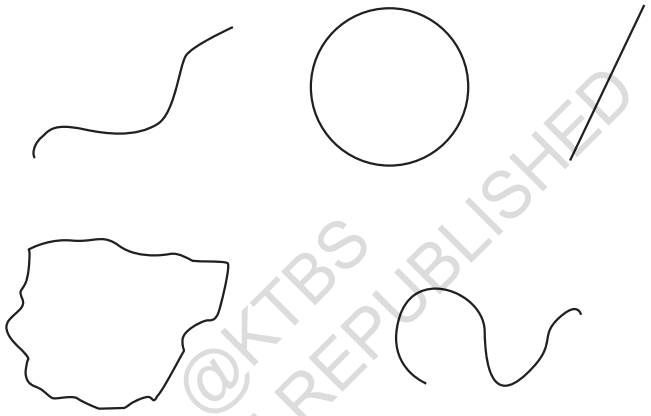


Fig. 8.1

Now, arm yourself with a ruler and a compass. Let us explore if we can draw these figures with these tools and get familiar with a compass.

Observe the way a compass is made. What can one draw with the compass? Explore!

Do you know what curves are? They are any shapes that can be drawn on paper with a pencil, and include straight lines, circles and other figures as shown below:



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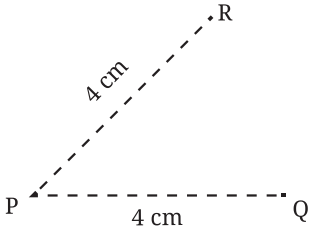
Mark a point 'P' in your notebook. Then, mark as many points as possible, in different directions, that are 4 cm away from P.

Think: Imagine marking all the points of 4 cm distance from the point P. How would they look?

Try to draw it and verify if it is correct by taking some points on the curve and checking if their distances from P are indeed 4 cm.

Explore, if you have not already done so, and see if a compass can be used for this purpose.

You can start by marking a few points of distance 4 cm from P using the compass. How can this be done?



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You will have to open up the compass against a ruler (see Fig. 8.2) such that the distance between the tip of the compass and the pencil is 4 cm.

Now, try to get the full curve.

Hint: Keep the point of the compass fixed moving only the pencil.

What is the shape of the curve? It is a circle!

Take a point on the circle. What will be its distance from P—equal to 4 cm, less than 4 cm or greater than 4 cm? Similarly, what will be the distance between P and another point on the circle?

As shown in the figure, the point P is called the **centre** of the circle and the distance between the centre and any point on the circle is called the **radius** of the circle.

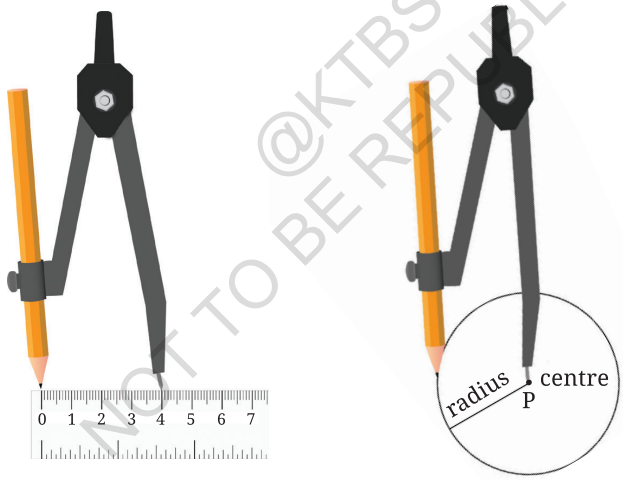


Fig. 8.2

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Having explored the use of a compass, go ahead and recreate the images in Fig. 8.1.

Can you make the figures look as good as the figures shown there? Try again if you want to!

Also, has the use of instruments made the construction easier?

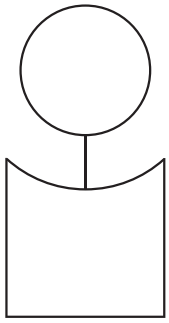
Now try constructing the following figures.

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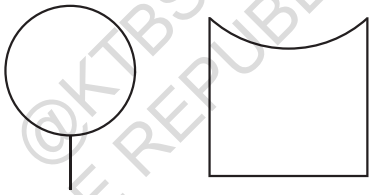
Construct

1. A Person

How will you draw this?

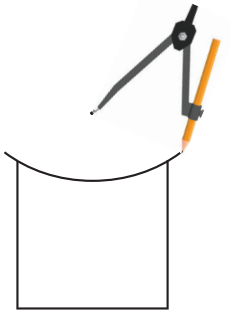


This figure has two components.



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You might have figured out a way of drawing the first part. For drawing the second part, see this.



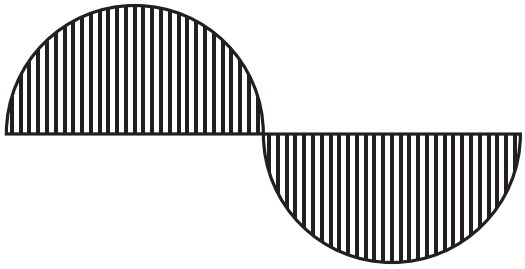
The challenge here is to find out where to place the tip of the compass and the radius to be taken for drawing this curve. You can fix a radius in the compass and try placing the tip of the

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compass in different locations to see which point works for getting the curve. Use your estimate where to keep the tip.

2. Wavy Wave

Construct this.

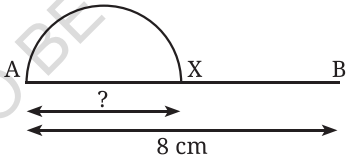


As the length of the central line is not specified, we can take it to be of any length.

Let us take AB to be the central line such that the length of AB is 8 cm. We write this as AB = 8 cm.

Here, the first wave is drawn as a half circle.





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Figure it Out

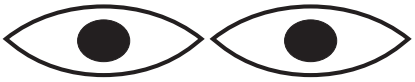
1. What radius should be taken in the compass to get this half circle? What should be the length of AX?
2. Take a central line of a different length and try to draw the wave on it.
3. Try to recreate the figure where the waves are smaller than a half circle (as appearing in the neck of the figure, 'A Person'). The challenge here is to get both the waves to be identical. This may be tricky!

Try This

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3. Eyes

How do you draw these eyes with a compass?



For a hint, go to the end of the chapter.

Make other artwork of your choice with a ruler and a compass.

8.2 Squares and Rectangles

Now, let us look at some basic figures having straight lines in their boundary.

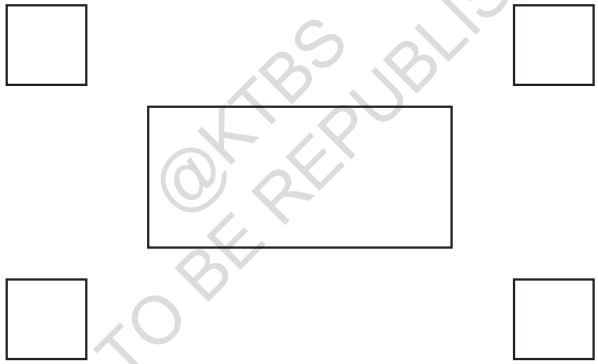


Fig. 8.3

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What shapes are these? Yes, these are our familiar squares and rectangles. But what makes them squares and rectangles?

Consider this rectangle ABCD.

The points A, B, C and D are the corners of the rectangle. Lines AB, BC, CD and DA are its sides. Its angles are ∠A, ∠B, ∠C and ∠D.

The blue sides AB and CD are called **opposite sides**, as they lie opposite to each other. Likewise, AD and BC is the other pair of opposite sides.

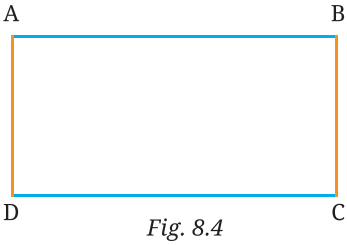


Fig. 8.4

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Recall that, in a rectangle:

R1) The opposite sides are equal in length, and

R2) All the angles are 90°.

As in the case of rectangles, the corners and sides are defined for a square in the same manner.

A square satisfies the following two properties:

S1) All the sides are equal, and

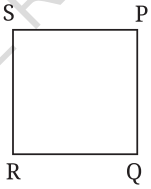
S2) All the angles are 90°.

See the rectangle in Fig. 8.4 and the name given to it: ABCD. This rectangle can also be named in other ways—BCDA, CDAB, DABC, ADCB, DCBA, CBAD and BADC. So, can a rectangle be named using any combination of the labels around its corners? No! For example, it cannot be named ABDC or ACBD. Can you see what names are allowed and what names are not?

In a valid name, the corners occur in an order of travel around the rectangle, starting from any corner.

Which of the following is not a name for this square?

1. PQSR
2. SPQR
3. RSPQ
4. QRSP



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Rotated Squares and Rectangles

Here is a square piece of paper having all its sides equal in length and all angles equal to 90°. It is rotated as shown in the figure. Is it still a square?

Let us check if the rotated paper still satisfies the properties of a square.

- Are all the sides still equal? Yes.
- Are all the angles still 90°? Yes.

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Rotating a square does not change its lengths and angles.

Therefore, this rotated figure satisfies both the properties of a square and so, it is a square.

By the same reasoning, a rotated rectangle is still a rectangle.

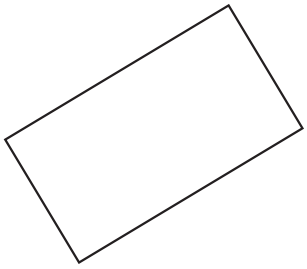
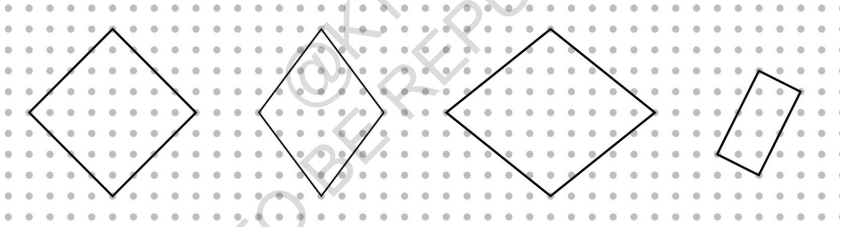


Figure it Out

1. Draw the rectangle and four squares configuration (shown in Fig. 8.3) on a dot paper.

   What did you do to recreate this figure so that the four squares are placed symmetrically around the rectangle? Discuss with your classmates.

2. Identify if there are any squares in this collection. Use measurements if needed.



   Think: Is it possible to reason out if the sides are equal or not, and if the angles are right or not without using any measuring instruments in the above figure? Can we do this by only looking at the position of corners in the dot grid?

3. Draw at least 3 rotated squares and rectangles on a dot grid. Draw them such that their corners are on the dots. Verify if the squares and rectangles that you have drawn satisfy their respective properties.

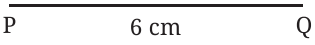
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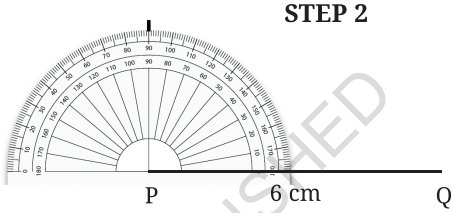
8.3 Constructing Squares and Rectangles

Now, let us start constructing squares and rectangles. How would you construct a square with a side of 6 cm?
For help, you can see the following figures. A square PQRS of side length 6 cm is constructed.

STEP 1



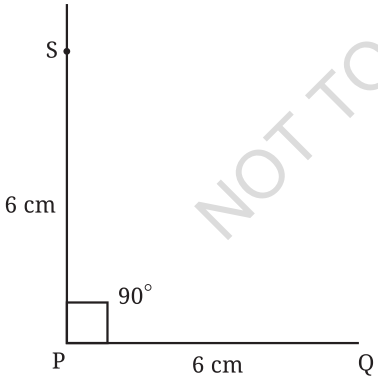
STEP 2



Mark a point to draw a perpendicular to PQ through P.

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STEP 3
Method 1



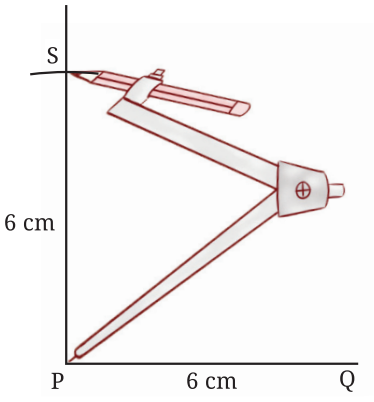
Mark S on the perpendicular such that PS = 6 cm using a ruler.

Method 2

This can also be done using a compass.

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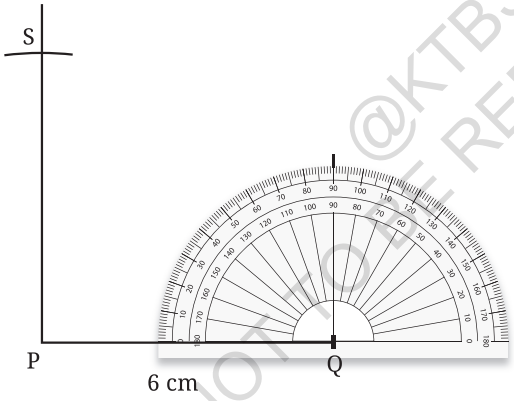




Can you see why PS should be 6 cm long?

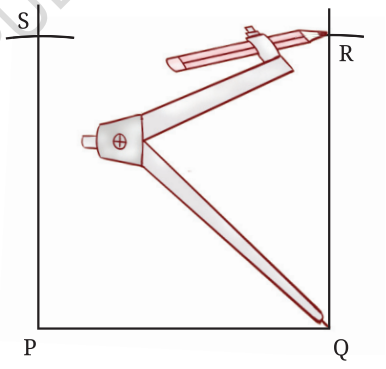
STEP 4

Draw a perpendicular to line segment PQ through Q.

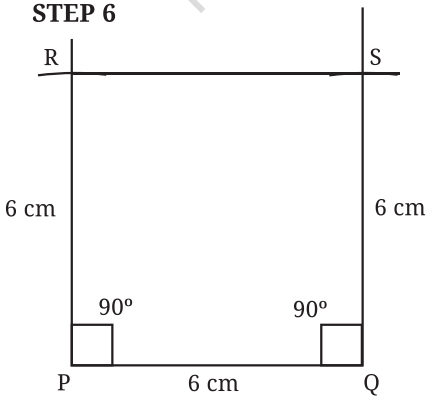


STEP 5

If we had used the compass, then the next point can easily be marked using it!



STEP 6



How long is the side RS and what are the measures of ∠R and ∠S?

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Construct

1. Draw a rectangle with sides of length 4 cm and 6 cm. After drawing, check if it satisfies both the rectangle properties.
2. Draw a rectangle of sides 2 cm and 10 cm. After drawing, check if it satisfies both the rectangle properties.
3. Is it possible to construct a 4-sided figure in which—
   - all the angles are equal to 90° but
   - opposite sides are not equal?

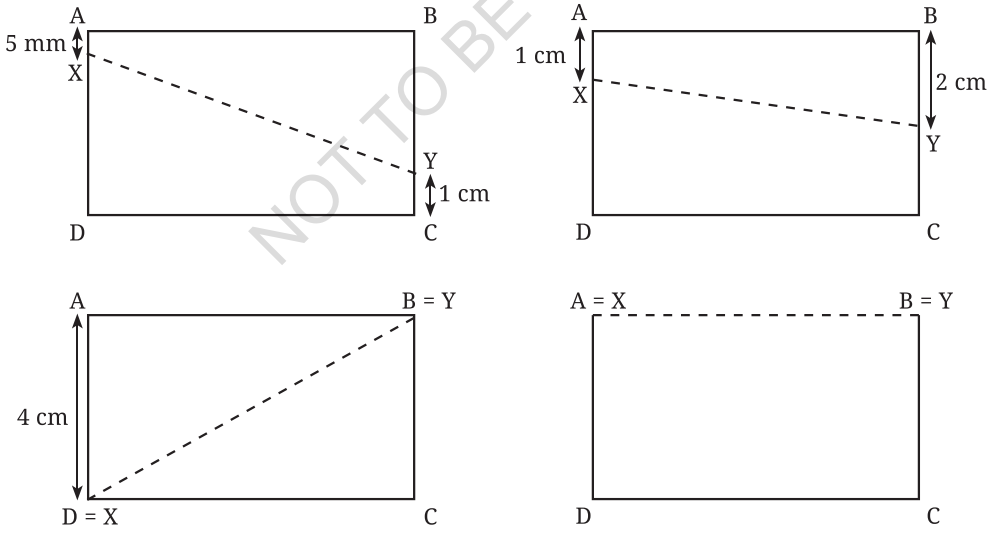
Try This

8.4 An Exploration in Rectangles

Construct a rectangle ABCD with AB = 7 cm and BC = 4 cm.

Imagine X to be a point that can be moved anywhere along the side AD. Similarly, imagine Y to be a point that can be moved anywhere along the side BC. Note that X can also be placed on the end point A or D. Similarly, Y can also be placed on the end point B or C.

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At which positions will the points X and Y be at their closest? When do you think they will be the farthest? What does your intuition say? Discuss with your classmates.

Math Talk

Now, verify your guesses by placing the points X and Y on the sides and measure how near or far they are.

The distance between X and Y can be obtained by measuring the length of the line XY.

How does the minimum distance between the points X and Y compare to the length of AB?

Change the positions of X and Y to check if there are other positions where they are at their nearest or farthest. You could construct multiple copies of the rectangle and try out various positions of X and Y.

How will you keep track of the lengths XY for different positions of X and Y?

Here is one way of doing it. Suppose here are some of the positions of X and Y that you have considered:

- When X is 5 mm away from A and Y is 3 cm away from B, XY = ___ cm __ mm
- When X is 1 cm away from A and Y is 1 cm away from B, XY = ___ cm ___ mm
- When X is 2 cm away from A and Y is 4 cm away from B, XY = ___ cm ___ mm and so on.

Is there *a* shorthand way of writing it down? In all the sentences, only the position of X, Y and the length XY changes. So we could write this as:

Distance of X from A	Distance of Y from B	Length of XY

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Have you checked what happens to the length XY when X and Y are placed at the same distance away from A and B, respectively? For example, as in the cases like these:

Distance of X from A	Distance of Y from B	Length of XY
5 mm	5 mm	
1 cm	1 cm	
1 cm 5 mm	1 cm 5 mm	

and so on.

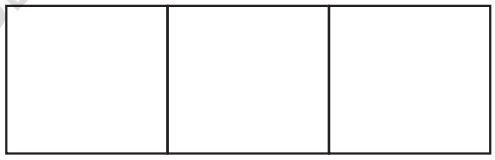
In each of these cases, observe
1. how the length XY compares to that of AB and
2. the shape of the 4-sided figure ABYX.

How does the farthest distance between X and Y compare with the length of AC? BD?

Construct

Breaking Rectangles

Construct a rectangle that can be divided into 3 identical squares as shown in the figure.



Solution

If this seem difficult, let us simplify the problem.

Explore

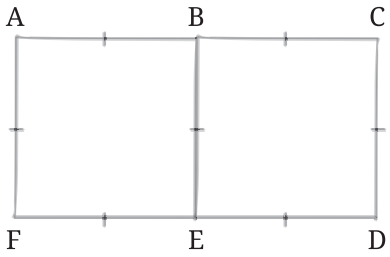
What about constructing a rectangle that can be divided into two identical squares? Can you try it?

It is wise to first plan and then construct. But how do we plan? Can you think of a way?

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One way is to visualise the final figure by drawing a rough diagram of it.



What can we infer from this figure?

Can you identify the equal sides?

Since, the two squares are identical,

AB = BC and FE = ED

Since ABEF and BCDE are squares, all the sides in each of the squares are equal. This is written as—

AF = AB = BE = FE

BE = BC = CD = ED

So, all the shorter lines are equal!

A convention is followed to represent equal sides. It is done by putting a '|' on the line. Refer to the rough figure.

Using this analysis, can you try constructing it? Remember, all that was asked for is a rectangle that can be divided into two identical squares and with no measurements imposed.

To draw the rectangle *ACDF*, one could assign any length to *AF*. For example, if we assign *AF* = 4 cm, then what must the length of *AC* be?

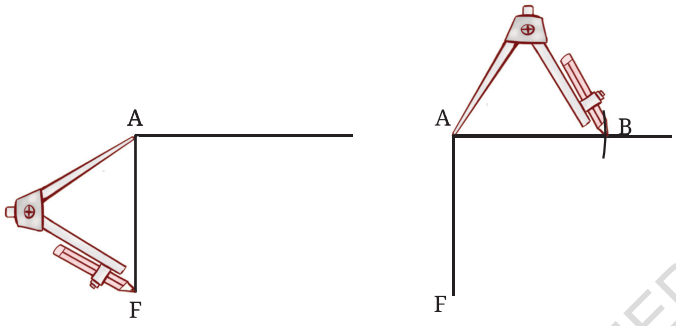
Explore: Can the rectangle now be completed?

In fact, one could proceed by drawing AF without even measuring its length using a ruler. We could then construct a line perpendicular to AF that is long enough to contain the other side. As, AB = AF, we need to somehow transfer the length of AF

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to get the point B. How do we do it without a ruler? Can it be done using a compass?
Observe, how the length of AF is measured using a compass.



Use it to mark out the points B and C, and complete the rectangle.

With this idea, try constructing a rectangle that can be divided into three identical squares.

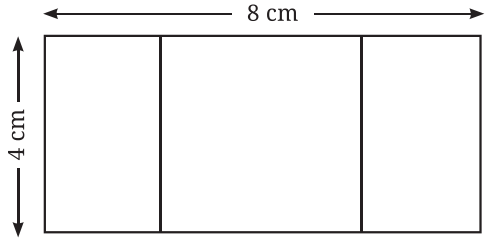
Give the lengths of the sides of a rectangle that cannot be divided into—

- two identical squares;
- three identical squares.

Construct

1. A Square within a Rectangle

Construct a rectangle of sides 8 cm and 4 cm. How will you construct a square inside, as shown in the figure, such that the centre of the square is the same as the centre of the rectangle?

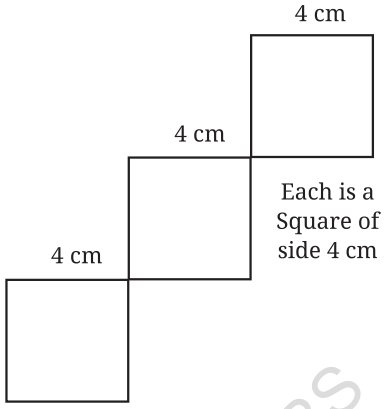


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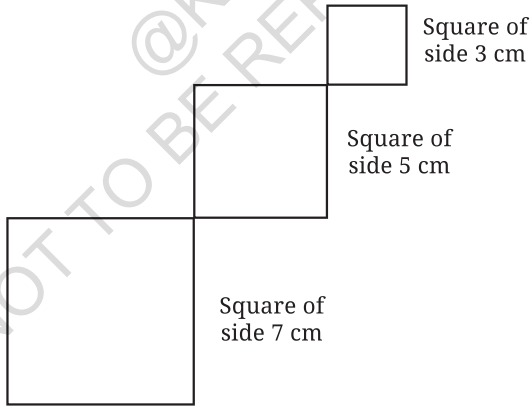
Hint: Draw a rough figure. What will be the sidelength of the square? What will be the distance between the corners of the square and the outer rectangle?

2. Falling Squares



Make sure that the squares are aligned the way they are shown.

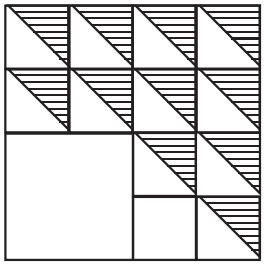
Now, try this.



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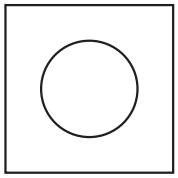
3. Shadings

Construct this. Choose measurements of your choice. Note that the larger 4-sided figure is a square and so are the smaller ones.



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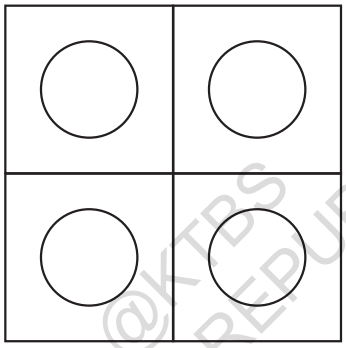
4. Square with a Hole



Observe that the circular hole is the same as the centre of the square.

Hint: Think where the centre of the circle should be.

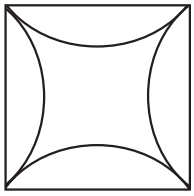
5. Square with more Holes



6. Square with Curves

This is a square with 8 cm sidelengths.

Hint: Think where the tip of the compass can be placed to get all the 4 arcs to bulge uniformly from each of the sides. Try it out!



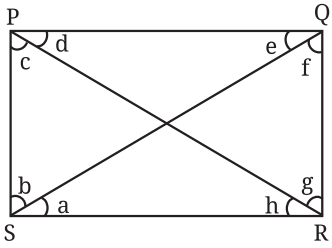


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8.5 Exploring Diagonals of Rectangles and Squares

Consider a rectangle PQRS. Join PR and QS. These two lines are called the **diagonals** of the rectangle.

Compare the lengths of the diagonals. First predict the answer. Then construct a



rectangle marking the points as shown and measure the diagonals. In rectangle PQRS, the right angles at P and R are referred to as opposite angles. The other pair of opposite angles are the right angles at Q and S.

Observe that a diagonal divides each of the pair of opposite angles into two smaller angles. In the figure, the diagonal PR divides angle R into two smaller angles which we simply call g and h. The diagonal also divides angle P into c and d. Are g and h equal? Are c and d equal?

First predict the answers, and then measure the angles. What do you observe? Identify pairs of angles that are equal.

Explore

How should the rectangle be constructed so that the diagonal divides the opposite angles into equal parts?

How will you record your observations? First, identify the parameters that need to be tracked. They are the sides of the rectangle and the 8 angles formed by the two diagonals. Are there any other measurements that you would want to keep track of?

Sides	A	B	C	D	E	F	G	H

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In your experimentation, did you consider the case when all four sides of the rectangle are equal? That is, did you consider the case of a square? See what happens in this special case!

Math Talk

What general laws did you observe with respect to the angles and sides? Try to frame and discuss them with your classmates.

How can one be sure if the laws that you have observed will always be true?

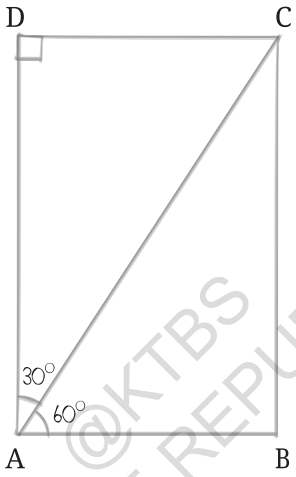
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Construct

1. Construct a rectangle in which one of the diagonals divides the opposite angles into 60° and 30°.

Solution

Let us start with a rough diagram.



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In what order should its parts be drawn?

We will briefly sketch a possible order of construction.

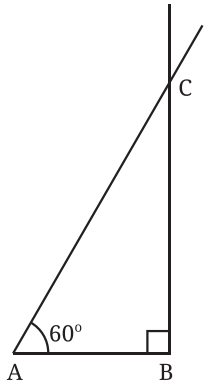
STEP 1



AB is drawn with an arbitrary length. What is the next point that can be located?

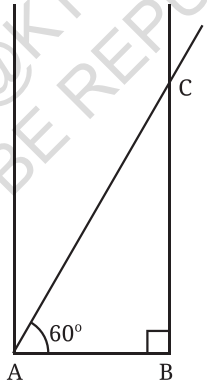
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STEP 2



STEP 3

We know the line on which D lies. Draw a line through A perpendicular to AB.



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Now ∠A is divided into two angles. One measures 60°. Check what the other angle is.

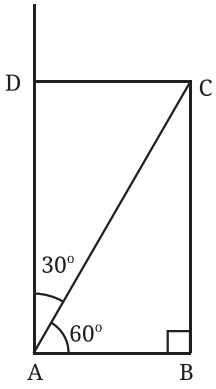
There are at least two ways of finding the point D—

- One uses the fact that all the angles of a rectangle are right angles.
- The other uses the fact that opposite sides are equal.

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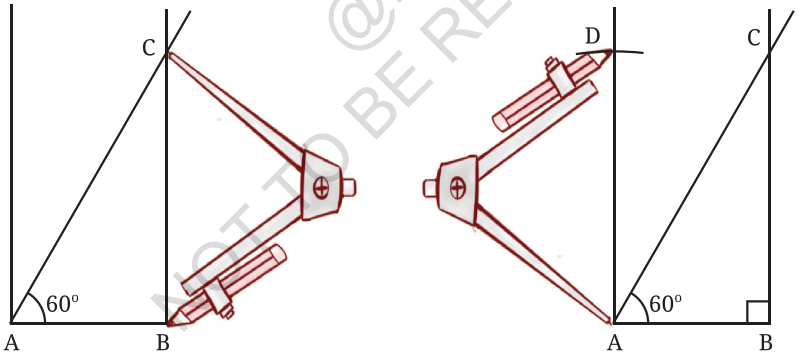
STEP 4

Method 1



Draw a line perpendicular to BC at C to get the point D.

Method 2



Using a compass, mark the point D such that AD = BC.
Join CD to get the required rectangle.

We have seen how to construct rectangles when their sides are given. But what do we do if a side and a diagonal is given?

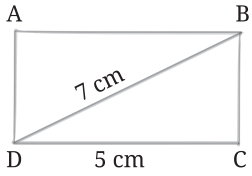
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2. Construct a rectangle where one of its sides is 5 cm and the length of a diagonal is 7 cm.

Solution

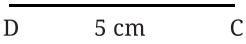
Let us draw a rough diagram.



Let us decide the steps of construction. Which line can be drawn first?

STEP 1

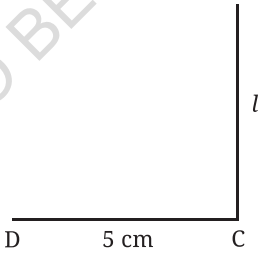
The base CD measuring length 5 cm can be easily constructed.



Next?

STEP 2

Draw a perpendicular to line DC at the point C. Let us call this line *l*.



This is easy as we know that this line is perpendicular to the base. The point B should be somewhere on this line *l*.

How do we spot it? What else do we know about the position of B?

We know that it is at a distance of 7 cm from the point D.

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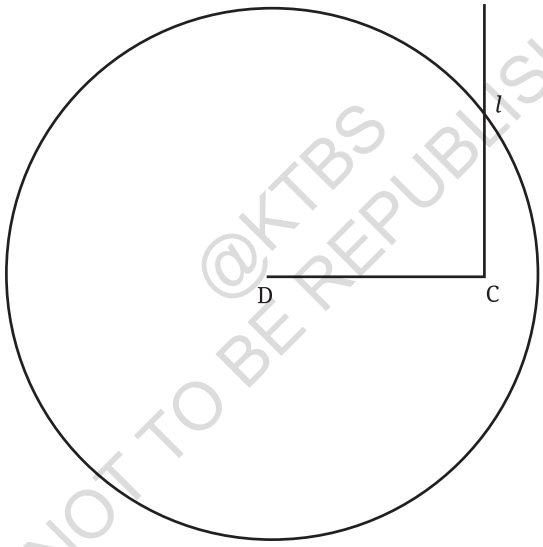
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One of the ways of marking B is by taking a ruler and trying to move it around to get a point on line l that is 7 cm from point D. However, this requires trial and error. There is another efficient method which doesn't involve trial and error.

For this, instead of trying to get that one required point of distance 7 cm from D, let us explore a way of getting all the points of distance 7 cm from D.

We know what this shape is!

STEP 3
Method 1



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Construct a circle of radius 7 cm with point D as the centre.

Can you spot the point B here? Remember that it is 7 cm away from point D and on the line l.

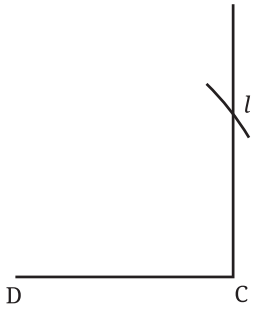
Consider the point at which the circle and the line intersect. What is its distance from point D? If needed, check your figure. What do you observe?

The point where the circle intersects the line l is the required point B.

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Method 2

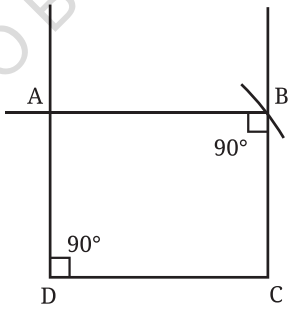
To locate the point B, was it necessary to draw the entire circle? We can see that only the arc near the line l is needed. So, the third step can also be done as shown in the figure below.



Having marked the three points of the rectangle, we only need to complete it. Recall that we were in a similar situation in the previous problem also. We saw two methods of completing the rectangle from here. We could follow any one of those methods.

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STEP 4



Construct perpendiculars to DC and BC passing through D and B, respectively. The point where these lines intersect is the fourth point A.

Check if ABCD is indeed a rectangle satisfying properties R1 and R2.

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Construct

1. Construct a rectangle in which one of the diagonals divides the opposite angles into 50° and 40°.
2. Construct a rectangle in which one of the diagonals divides the opposite angles into 45° and 45°. What do you observe about the sides?
3. Construct a rectangle one of whose sides is 4 cm and the diagonal is of length 8 cm.
4. Construct a rectangle one of whose sides is 3 cm and the diagonal is of length 7 cm.

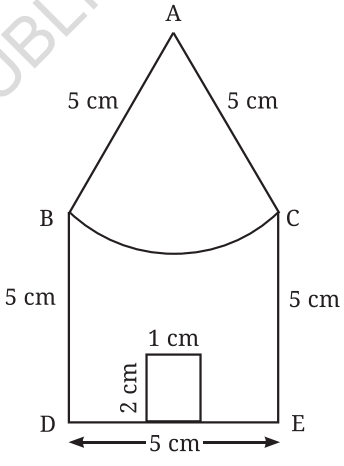
8.6 Points Equidistant from Two Given Points

Construct

House

Recreate this figure.

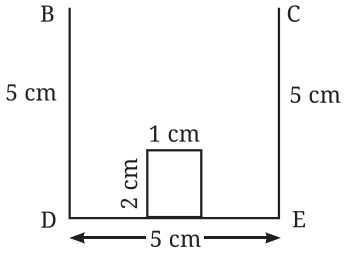
Note that all the lines forming the border of the house are of length 5 cm.



Solution

The first task is to identify in what sequence the lines and curve will have to be drawn.

STEP 1



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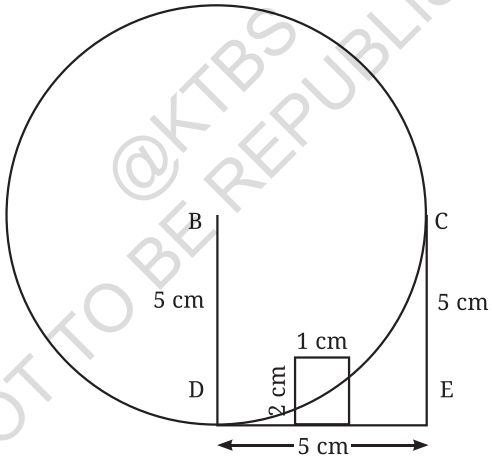
Can you complete the figure? Try!

We need to locate the point A that is of distance 5 cm from the points B and C. You might have realised that this can be done using a ruler. However, this leads to a lot of trial and error. This construction can be further simplified. How?

If you have guessed that this can be done by the use of compass, you are right! Go ahead and explore how the point A can be located without trial and error.

There is a similarity between the problem of finding point A in this problem and point B at step 3 of the second solved example of the previous section (see page 209).

STEP 2



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Draw a curve that has all its points of 5 cm from the point B; the circle centred at B should be with 5 cm radius.

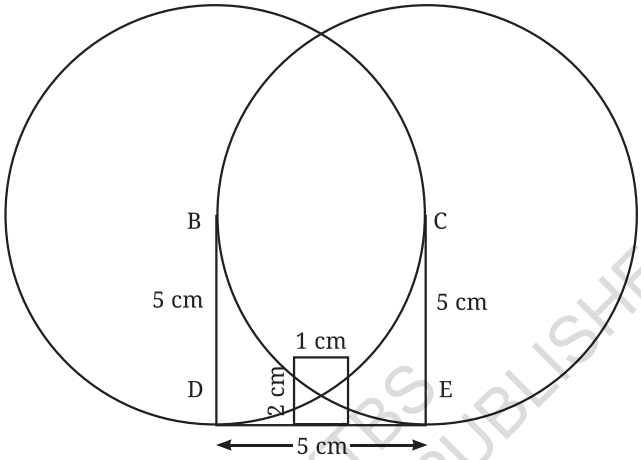
Does this help in spotting the point A? Construct and explore in the figure.

The point A can be located by finding the correct point on the circle that is of distance 5 cm from the point C. Again, this can be done using a ruler. But can we use a compass for this?

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STEP 3
Method 1

Take a radius of 5 cm in the compass and with C as the centre, draw a circle.



Are you able to spot the point A? Check the figure in your notebook. What do you observe?

See the point at which both the circles intersect. How far is it from the point B?

How far is it from C?

Thus, this is the point A!

Think

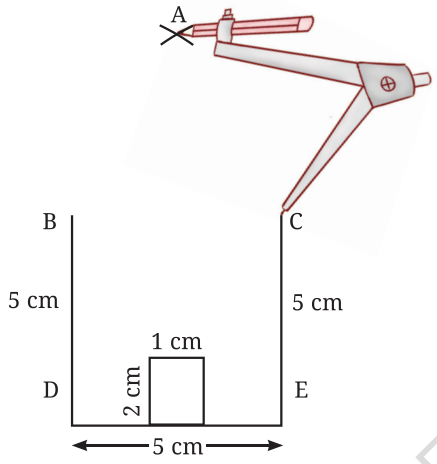
Was it necessary to draw two full circles to get the point A? We only needed part of both the circles.

Method 2

So the point A could have been obtained just by drawing arcs of radius 5 cm from points B and C.

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Join A to B and A to C by straight lines.

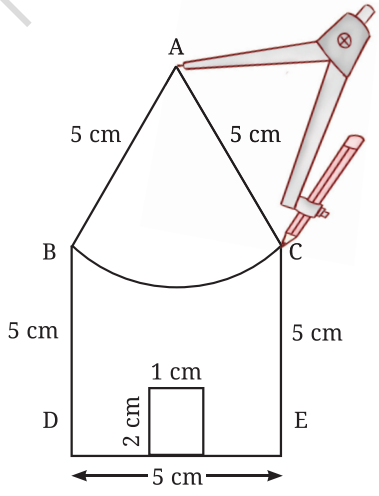
Having obtained point A, what remains is the construction of the remaining arc. How do we do it?

Can we use the fact that A is of distance 5 cm from both B and C?

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STEP 4

Take 5 cm radius in the compass and from A, draw the arc touching B and C as shown in the figure.



The house is ready!

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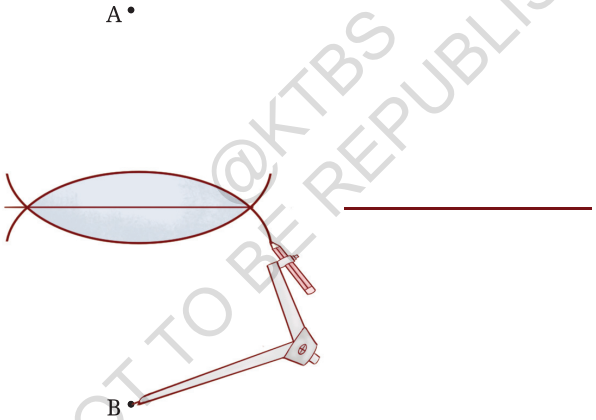
Construct

1. Construct a bigger house in which all the sides are of length 7 cm.
2. Try to recreate 'A Person', 'Wavy Wave', and 'Eyes' from the section 'Artwork', using ideas involved in the 'House' construction.
3. Is there a 4-sided figure in which all the sides are equal in length but is not a square? If such a figure exists, can you construct it?

Hints

A) Eyes (from 8.1 Artwork and Construct above (page no. 215).

Part of the construction is shown earlier. Observe it carefully. You will see two horizontal lines drawn lightly. In geometric constructions, one often constructs supporting curves or figures that are not part of the given figure but help in constructing it.



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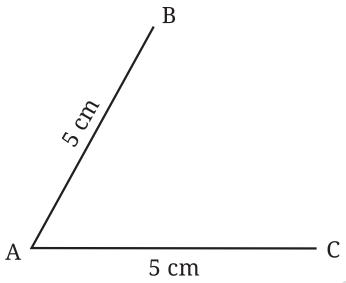
The technique to draw the upper and the lower curves of the eye is the same as that used in the figure, 'A Person'. Points A and B are the locations where the tip of the compass is placed when drawing the curves of the eye. Note that the upper curve and the lower curve should together form a symmetrical figure. For this to happen, where should these points A and B be placed? Make a good estimate.

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Try to get the eyes as symmetrical and identical as possible. This might need many trials.

B) (From Construct 3 above).

For the purpose of construction, let us take the side lengths to be of 5 cm. Consider this figure.



We need to identify only one more point to make this a 4-sided figure. That point, let us call it D, should be 5 cm from both B and C. How can such a point be found?
Can any of the ideas used in the 'House' problem be used here?

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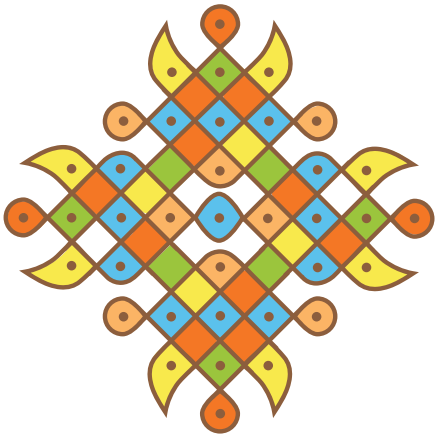
SUMMARY

- All the points of a circle are at the same distance from its **centre**. This distance is called the **radius** of the circle.
- A compass can be used to construct circles and their parts.
- A rough diagram can be useful in planning how to construct a given figure.
- A rectangle can be constructed given the lengths of its sides or that of one of its sides and a diagonal.

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9 SYMMETRY

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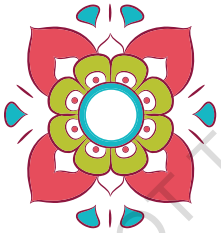
Look around you—you may find many objects that catch your attention. Some such things are shown below:



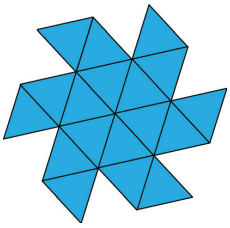
Flower



Butterfly



Rangoli



Pinwheel

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There is something beautiful about the pictures above.

The flower looks the same from many different angles. What about the butterfly? No doubt, the colours are very attractive. But what else about the butterfly appeals to you?

In these pictures, it appears that some parts of the figure are repeated and these repetitions seem to occur in a definite pattern. Can you see what repeats in the beautiful *rangoli* figure? In the

rangoli, the red petals come back onto themselves when the flower is rotated by 90° around the centre and so do the other parts of the *rangoli*.

What about the pinwheel? Can you spot which pattern is repeating? *Hint*: Look at the hexagon first.

Now, can you say what figure repeats along each side of the hexagon? What is the shape of the figure that is stuck to each side? Do you recognise it? How do these shapes move as you move along the boundary of the hexagon? What about the other pictures—what is it about those structures that appeals to you and what are the patterns in those structures that repeat?



Clouds

On the other hand, look at this picture of clouds. There is no such repetitive pattern.

We can say that the first four figures are symmetrical and the last one is not symmetrical. A symmetry refers to a part or parts of a figure that are repeated in some definite pattern.

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Taj Mahal



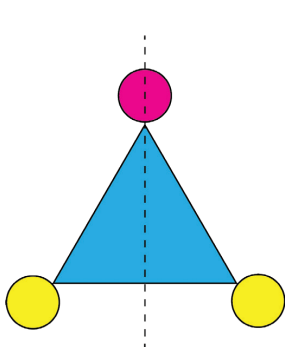
Gopuram

What are the symmetries that you see in these beautiful structures?

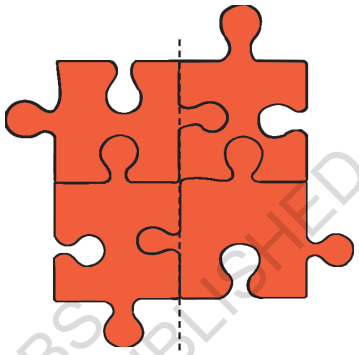
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9.1 Line of Symmetry

Figure (a) shows the picture of a blue triangle with a dotted line. What if you fold the triangle along the dotted line? Yes, one half of the triangle covers the other half completely. These are called **mirror halves**!



(a)



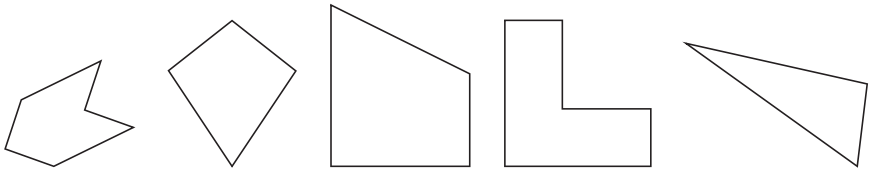
(b)

What about Figure (b) with the four puzzle pieces and a dotted line passing through the middle? Are they mirror halves? No, when we fold along the line, the left half does not exactly fit over the right half.

A line that cuts a figure into two parts that exactly overlap when folded along that line is called a **line of symmetry** of the figure.

Figure it Out

1. Do you see any line of symmetry in the figures at the start of the chapter? What about in the picture of the cloud?
2. For each of the following figures, identify the line(s) of symmetry if it exists.

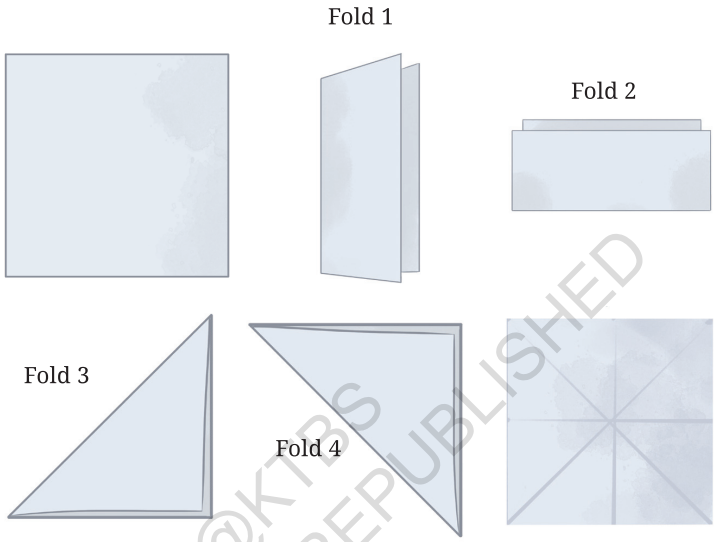


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Figures with more than one line of symmetry

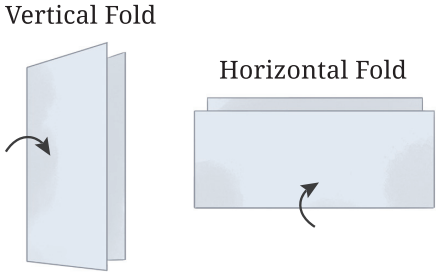
Does a square have only one line of symmetry?
Take a square piece of paper. By folding, find all its lines of symmetry.



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Here are the different folds giving different lines of symmetry.

- Fold the paper into half vertically.
- Fold it again into half horizontally (i.e., you have folded it twice). Now open out the folds.



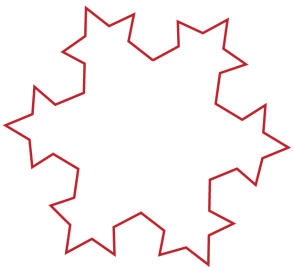
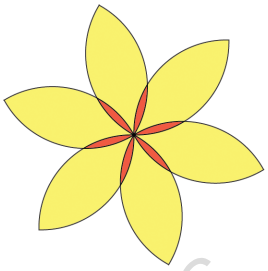
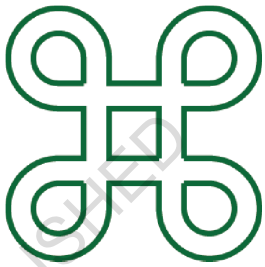
Again fold the square into half (for a third time now), but this time along a diagonal, as shown in the figure. Again, open it.

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Fold it into half (for the fourth time), but this time along the other diagonal, as shown in the figure. Open out the fold.

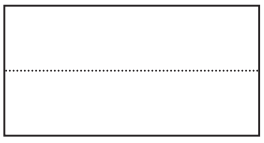
Is there any other way to fold the square so that the two halves overlap? How many lines of symmetry does the square shape have?

Thus, figures can have multiple lines of symmetry. The figures below also have multiple lines of symmetry. Can you find them all?

We saw that the diagonal of a square is also a line of symmetry. Let us take a rectangle that is not a square. Is its diagonal a line of symmetry?

First, see the rectangle and answer this question. Then, take a rectangular piece of paper and check if the two parts overlap by folding it along its diagonal. What do you observe?



Reflection

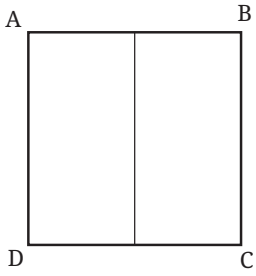
So far we have been saying that when we fold a figure along a line of symmetry, the two parts overlap completely. We could also say that the part of the figure on one side of the line of symmetry is reflected by the line to the other side; similarly, the part of the figure on the other side of the line of symmetry is reflected to the first side! Let us understand this by labeling some points on the figure.

The figure shows a square with its corners labeled A, B, C and D. Let us first consider the vertical line of symmetry. When we reflect

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the square along this line, the points B, C on the right get reflected to the left side and occupy the positions occupied earlier by A, D. What happens to the points A, D? A occupies the position occupied by B and D that of C!



What if we reflect along the diagonal from A to C? Where do points A, B, C and D go? What if we reflect along the horizontal line of symmetry?

A figure that has a line or lines of symmetry is thus, also said to have **reflection symmetry**.

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Generating shapes having lines of symmetry

So far we have seen symmetrical figures and asymmetrical figures. How does one generate such symmetrical figures? Let us explore this.

Ink Blot Devils

You enjoyed doing this earlier in Class 5. Take a piece of paper. Fold it in half. Open the paper and spill a few drops of ink (or paint) on one half.

Now press the halves together and then open the paper again.

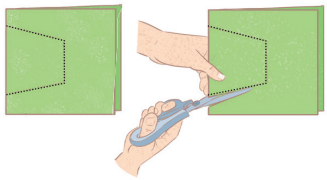
- What do you see?
- Is the resulting figure symmetric?
- If yes, where is the line of symmetry?
- Is there any other line along which it can be folded to produce two identical parts?
- Try making more such patterns.

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Paper Folding and Cutting

Here is another way of making symmetric shapes!

In these two figures, a sheet of paper is folded and a cut is made along the dotted line shown. Draw a sketch of how the paper will look when unfolded.

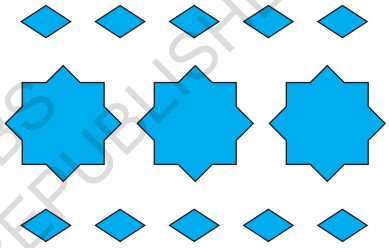


Do you see a line of symmetry in this figure? What is it?

Make different symmetric shapes by folding and cutting.

There are more ways of folding and cutting pieces of paper to get symmetric shapes!

Use thin rectangular coloured paper. Fold it several times and create some intricate patterns by cutting the paper, like the one shown here. Identify the lines of symmetry in the repeating design. Use such decorative paper cut-outs for festive occasions.

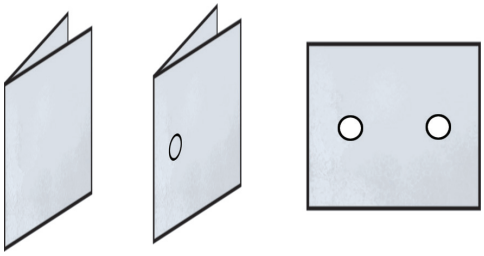


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Figure it Out

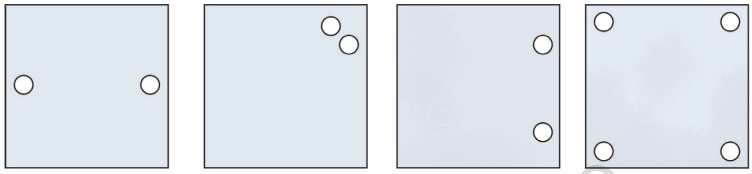
Punching Game

The fold is a line of symmetry. Punch holes at different locations of a folded square sheet of paper using a punching machine and create different symmetric patterns.



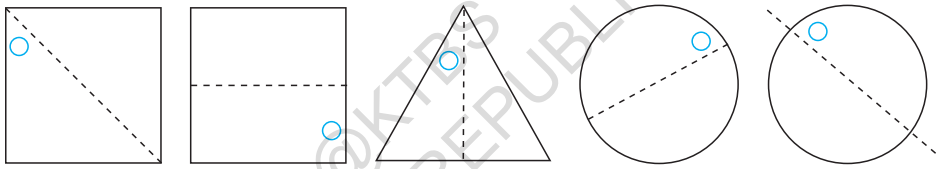
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1. In each of the following figures, a hole was punched in a folded square sheet of paper and then the paper was unfolded. Identify the line along which the paper was folded.
   Figure (d) was created by punching a single hole. How was the paper folded?



a. b. c. d.

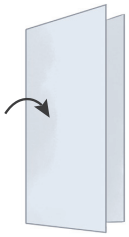
2. Given the line(s) of symmetry, find the other hole(s):



a. b. c. d. e.

3. Here are some questions on paper cutting.
   Consider a vertical fold. We represent it this way:

Vertical Fold



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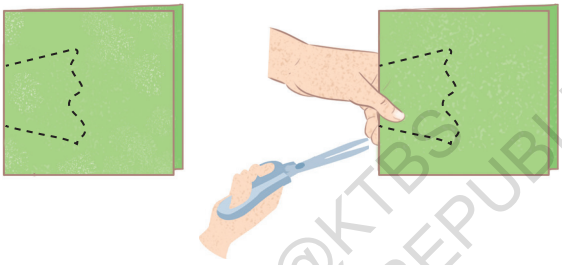
Similarly, a horizontal fold is represented as follows:

Horizontal Fold

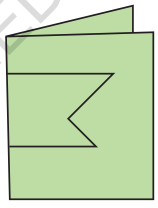


4. After each of the following cuts, predict the shape of the hole when the paper is opened. After you have made your prediction, make the cutouts and verify your answer.

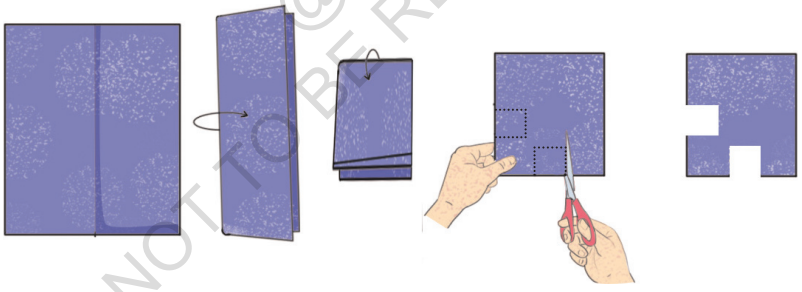
a.



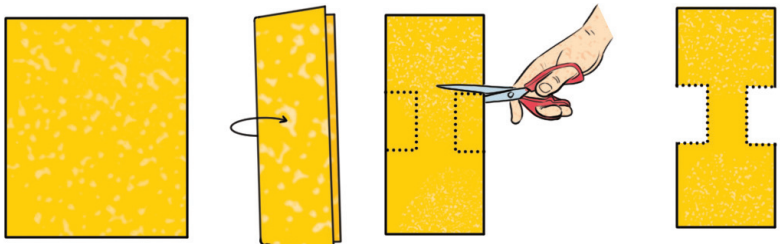
b.



c.



d.



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5 Suppose you have to get each of these shapes with some folds and a single straight cut. How will you do it?

a. The hole in the centre is a square.



b. The hole in the centre is a square.

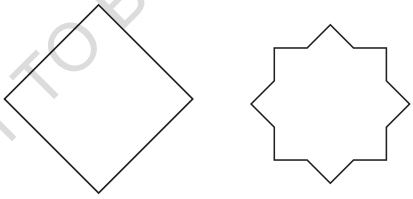


Note: For the above two questions, check if the 4-sided figures in the centre satisfy both the properties of a square.

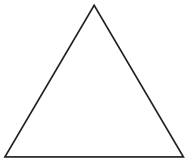
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6. How many lines of symmetry do these shapes have?

a.

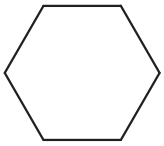


b. A triangle with equal sides and equal angles.

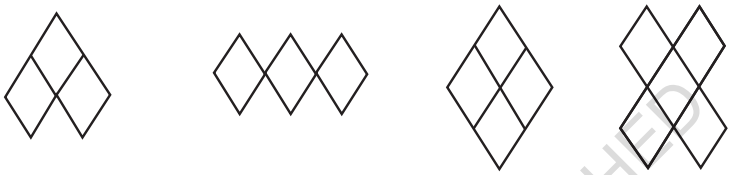


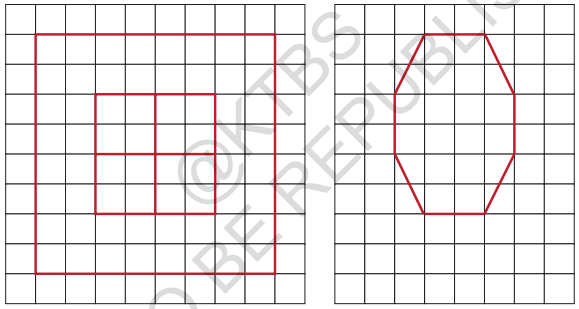
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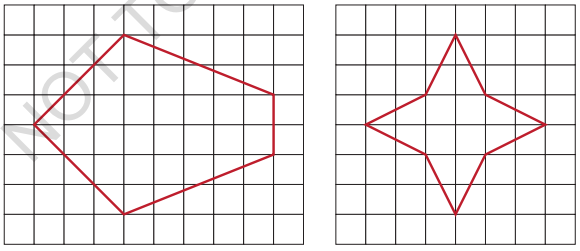
c. A hexagon with equal sides and equal angles.



7. Trace each figure and draw the lines of symmetry, if any:



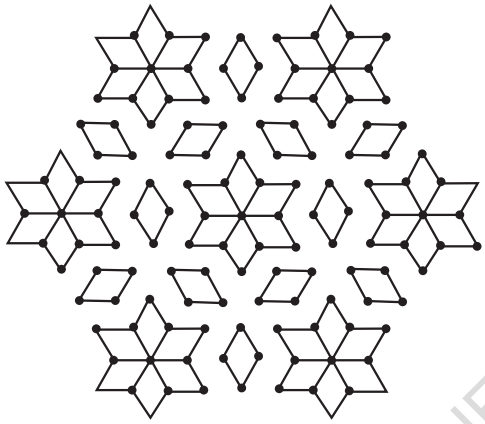




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8. Find the lines of symmetry for the *kolam* below.

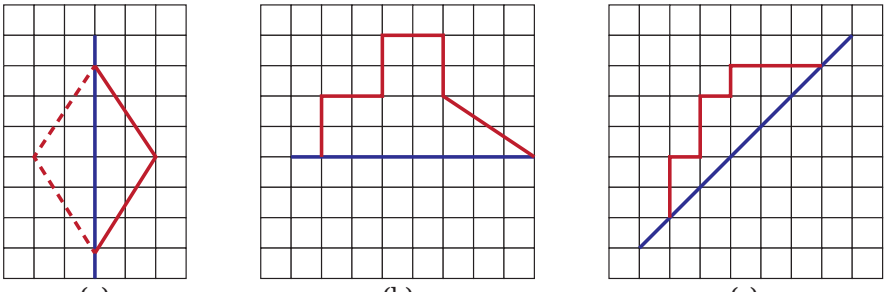


9. Draw the following.
   a. A triangle with exactly one line of symmetry.
   b. A triangle with exactly three lines of symmetry.
   c. A triangle with no line of symmetry.

   Is it possible to draw a triangle with exactly two lines of symmetry?

10. Draw the following. In each case, the figure should contain at least one curved boundary.
    a. A figure with exactly one line of symmetry.
    b. A figure with exactly two lines of symmetry.
    c. A figure with exactly four lines of symmetry.

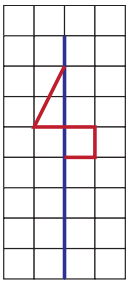
11. Copy the following on squared paper. Complete them so that the blue line is a line of symmetry. Problem (a) has been done for you.

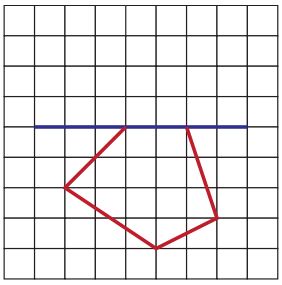


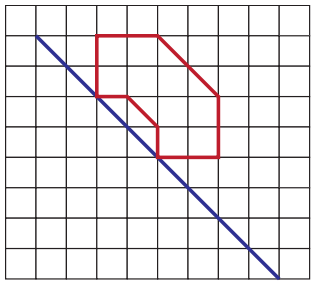
(a) (b) (c)

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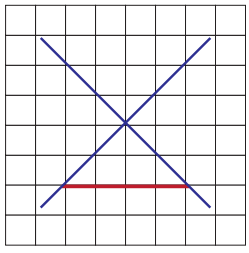

(d)

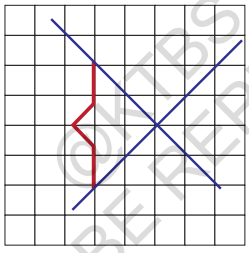

(e)

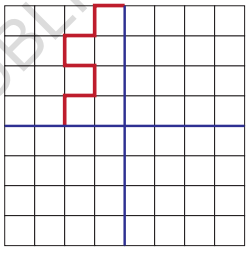

(f)

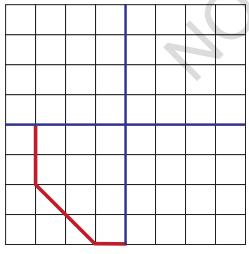
Hint: For (c) and (f), see if rotating the book helps!

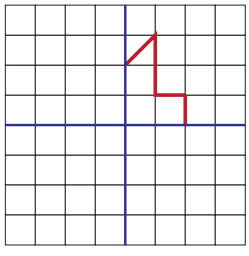
12. Copy the following drawing on squared paper. Complete each one of them so that the resulting figure has the two blue lines as lines of symmetry.

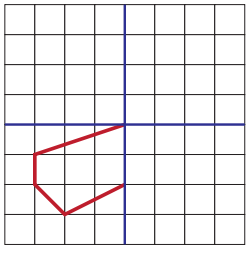

(a)


(b)


(c)


(d)

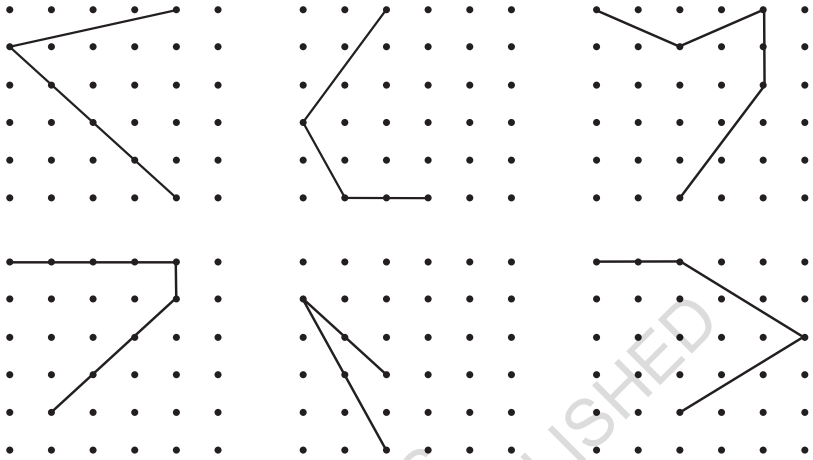

(e)


(f)

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13. Copy the following on a dot grid. For each figure draw two more lines to make a shape that has a line of symmetry.



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9.2 Rotational Symmetry



The paper windmill in the picture looks symmetrical but there is no line of symmetry! However, if you fold it, the two halves will not exactly overlap. On the other hand, if you rotate it by 90° about the red point at the centre, the windmill looks exactly the same.

We say that the windmill has **rotational symmetry**.

When talking of rotational symmetry, there is always a fixed point about which the object is rotated. This fixed point is called the **centre of rotation**.

Will the windmill above look exactly the same when rotated through an angle of less than 90°?

No!

An angle through which a figure can be rotated to look exactly the same is called an **angle of rotational symmetry**, or just an **angle of symmetry**, for short.

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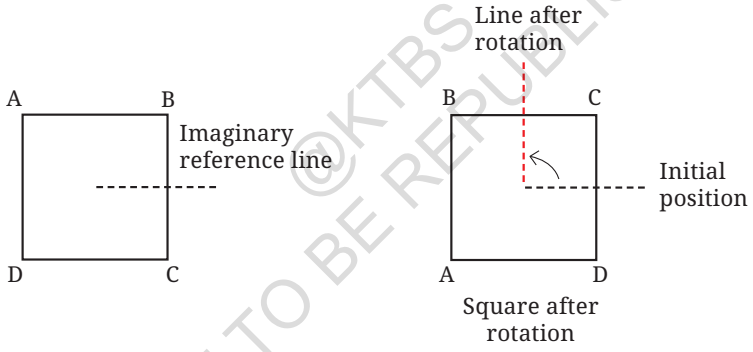
For the windmill, the angles of symmetry are 90° (quarter turn), 180° (half turn), 270° (three-quarter turn) and 360° (full turn). Observe that when any figure is rotated by 360°, it comes back to its original position, so 360° is always an angle of symmetry.

Thus, we see that the windmill has 4 angles of symmetry.

Do you know of any other shape that has exactly four angles of symmetry?

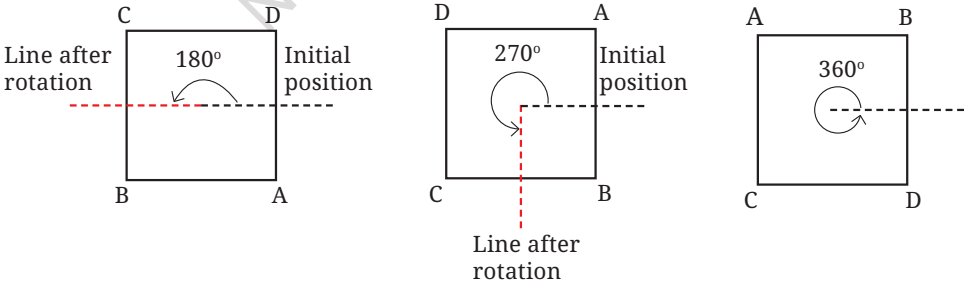
How many angles of symmetry does a square have? How much rotation does it require to get the initial square?

We get back a square overlapping with itself after 90° of rotation. This takes point A to the position of point B, point B to the position of point C, point C to the position of point D, and point D back to the position of point A. Do you know where to mark the centre of rotation?



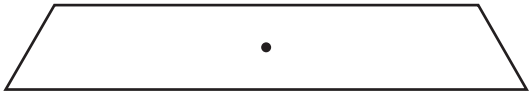
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What are the other angles of symmetry?

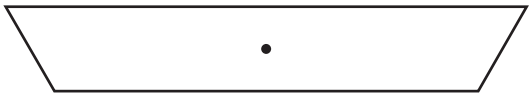


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Example: Find the angles of symmetry of the following strip.



Solution: Let us rotate the strip in a clockwise direction about its centre.



A rotation of 180° results in the figure above. Does this overlap with the original figure.

No. Why?

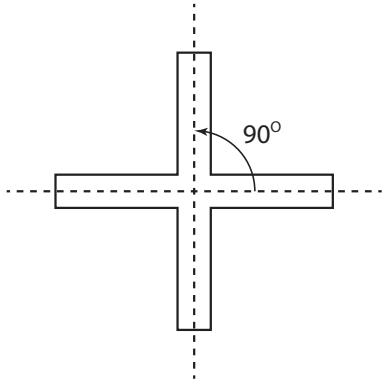
Another rotation through 180° from this position gives the original shape.

This figure comes back to its original shape only after **one** complete rotation through 360°. So, we say that this figure **does not have rotational symmetry**.

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Rotational Symmetry of Figures with Radial Arms

Consider this figure, a picture with 4 radial arms. How many angles of symmetry does it have? What are they? Note that the angle between adjacent central dotted lines is 90°.



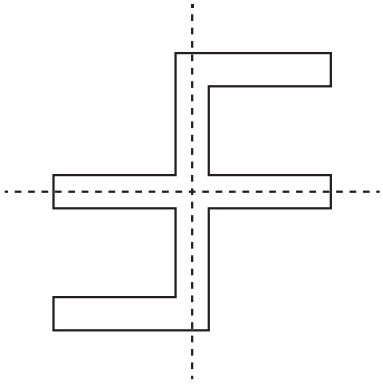
Can you change the angles between the radial arms so that the figure still has 4 angles of symmetry? Try drawing it.

To check if the figure drawn indeed has 4 angles of symmetry, you could draw the figure on two different pieces of paper. Cut out the radial arms from one of the papers. Keep the figure on the paper fixed and rotate the cutout to check for rotational symmetry.

How will you modify the figure above so that it has only two angles of symmetry?

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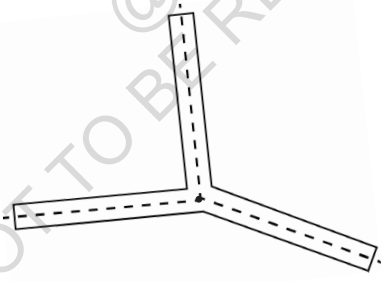
Here is one way:



We have seen figures having 4 and 2 angles of symmetry. Can we get a figure having exactly 3 angles of symmetry? Can you use radial arms for this?

Let us try with 3 radial arms as in the figure below. How many angles of symmetry does it have and what are they?

Here is a figure with three radial arms.



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Trace and cut out a copy of this figure. By rotating the cutout over this figure determine its angles of rotation.

We see that only a full turn or a rotation of 360° will bring the figure back into itself. So this figure does not have rotational symmetry as 360 degrees is its only angle of symmetry.

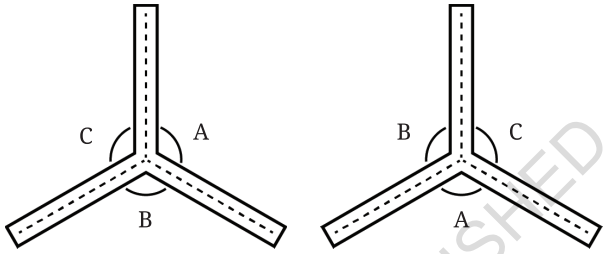
However, can anything in the figure be changed to make it have 3 angles of symmetry?

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Can it be done by changing the angles between the dotted lines?

If a figure with three radial arms should have rotational symmetry, then a rotated version of it should overlap with the original. Here are rough diagrams of both of them.

If these two figures must overlap, what can you tell about the angles?



Observe that ∠A must overlap ∠B, ∠B must overlap ∠C and ∠C must overlap ∠A.

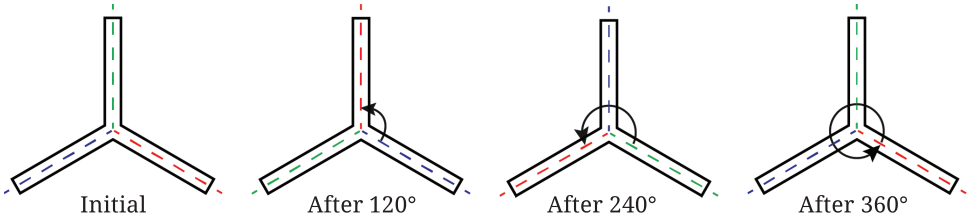
So, ∠A = ∠B = ∠C. What must this angle be?

We know that a full turn has 360 degrees. This is equally distributed amongst these three angles. So each angle must be $\frac{360°}{3}$ = 120°.

So, the radial arms figure with 3 arms shows rotational symmetry when the angle between the adjacent dotted lines is 120°. Use paper cutouts to verify this observation.

Now how many angles of rotation does the figure have and what are they?

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Note: The colours have been added to show the rotations.

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Let us explore more figures.

Can you draw a figure with radial arms that has a) exactly 5 angles of symmetry, b) 6 angles of symmetry? Also find the angles of symmetry in each case.

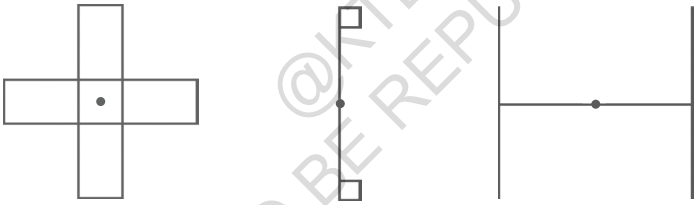
Hint: Use 5 radial arms for the first case. What should the angle between two adjacent radial arms be?

Consider a figure with radial arms having exactly 7 angles of symmetry. What will be its smallest angle of symmetry? Is the number of degrees a whole number in this case? If not, express it as a mixed fraction.

Let us find the angles of symmetry for other kinds of figures.

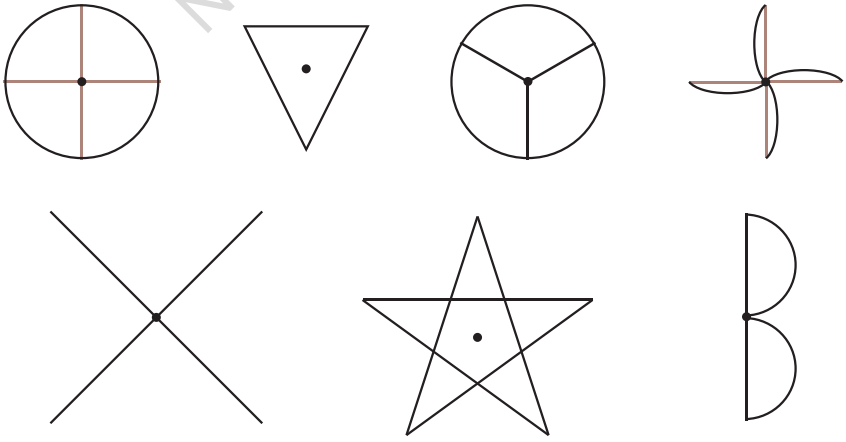
Figure it Out

1. Find the angles of symmetry for the given figures about the point marked •.



(a) (b) (c)

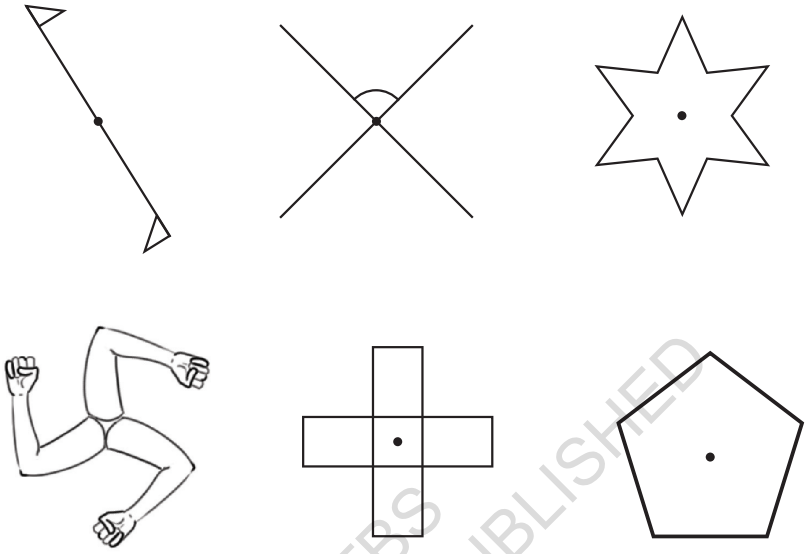
2. Which of the following figures have more than one angle of symmetry?



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3. Give the order of rotational symmetry for each figure:



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Let us list down the angles of symmetry for all the cases above.

- Angles of symmetry when there are exactly 2 of them: 180°, 360°.
- Angles of symmetry when there are exactly 3 of them: 120°, 240°, 360°.
- Angles of symmetry when there are exactly 4 of them: 90°, 180°, 270°, 360°.

Do you observe something common about the angles of symmetries in these cases? The first set of numbers are all multiples of 180. The second are all multiples of 120. The third are all multiples of 90.

In each case, the angles are the multiples of the smallest angle. You may wonder and ask if this will always happen. What do you think?

True or False

- Every figure will have 360 degrees as an angle of symmetry.

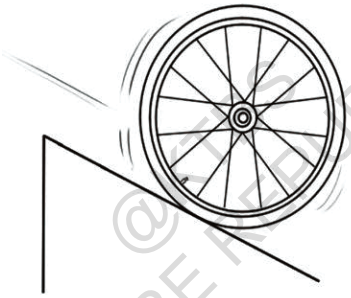
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- If the smallest angle of symmetry of a figure is a natural number in degrees, then it is **a factor** of 360.

Is there a smallest angle of symmetry for all figures? It turns out that this is the case for most figures, except for the most symmetric shapes like the circle, whose symmetries we now discuss.

Symmetries of a circle

The circle is a fascinating figure. What happens when you rotate a circle clockwise about its centre? It coincides with itself. It does not matter what angle you rotate it by! So, for a circle, every angle is an angle of symmetry.



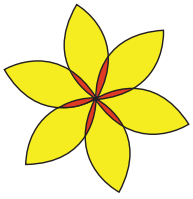
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Now take a point on the rim of the circle and join it to the centre. Extend the segment to a diameter of the circle. Is that diameter a line of reflection symmetry? It is. Every diameter is a line of symmetry!

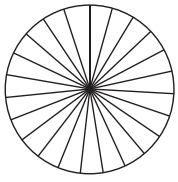
Like wheels, we can find other objects around us having rotational symmetry. Find them. Some of them are shown below:



Fan



Flower

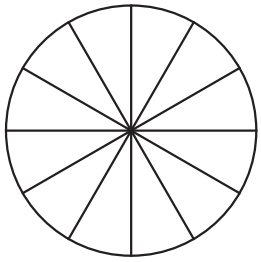


Wheel

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Figure it Out

1. Colour the sectors of the circle below so that the figure has i) 3 angles of symmetry, ii) 4 angles of symmetry, iii) what are the possible numbers of angles of symmetry you can obtain by colouring the sectors in different ways?



2. Draw two figures other than a circle and a square that have both reflection symmetry and rotational symmetry.
3. Draw, wherever possible, a rough sketch of:
   a. A triangle with at least two lines of symmetry and at least two angles of symmetry.
   b. A triangle with only one line of symmetry but not having rotational symmetry.
   c. A quadrilateral with rotational symmetry but no reflection symmetry.
   d. A quadrilateral with reflection symmetry but not having rotational symmetry.

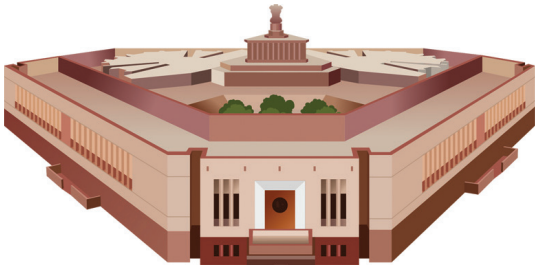
Try This

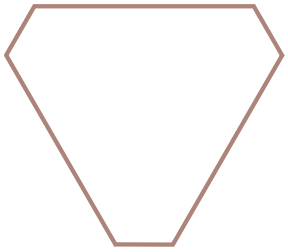
4. In a figure, 60° is the smallest angle of symmetry. What are the other angles of symmetry of this figure?
5. In a figure, 60° is an angle of symmetry. The figure has two angles of symmetry less than 60°. What is its smallest angle of symmetry?
6. Can we have a figure with rotational symmetry whose smallest angle of symmetry is:
   a. 45°?
   b. 17°?

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7. This is a picture of the new Parliament Building in Delhi.





   a. Does the outer boundary of the picture have reflection symmetry? If so, draw the lines of symmetries. How many are they?
   b. Does it have rotational symmetry around its centre? If so, find the angles of rotational symmetry.

8. How many lines of symmetry do the shapes in the first shape sequence in Chapter 1, Table 3, the Regular Polygons, have? What number sequence do you get?
9. How many angles of symmetry do the shapes in the first shape sequence in Chapter 1, Table 3, the Regular Polygons, have? What number sequence do you get?
10. How many lines of symmetry do the shapes in the last shape sequence in Chapter 1, Table 3, the Koch Snowflake sequence, have? How many angles of symmetry?
11. How many lines of symmetry and angles of symmetry does *Ashoka Chakra* have?

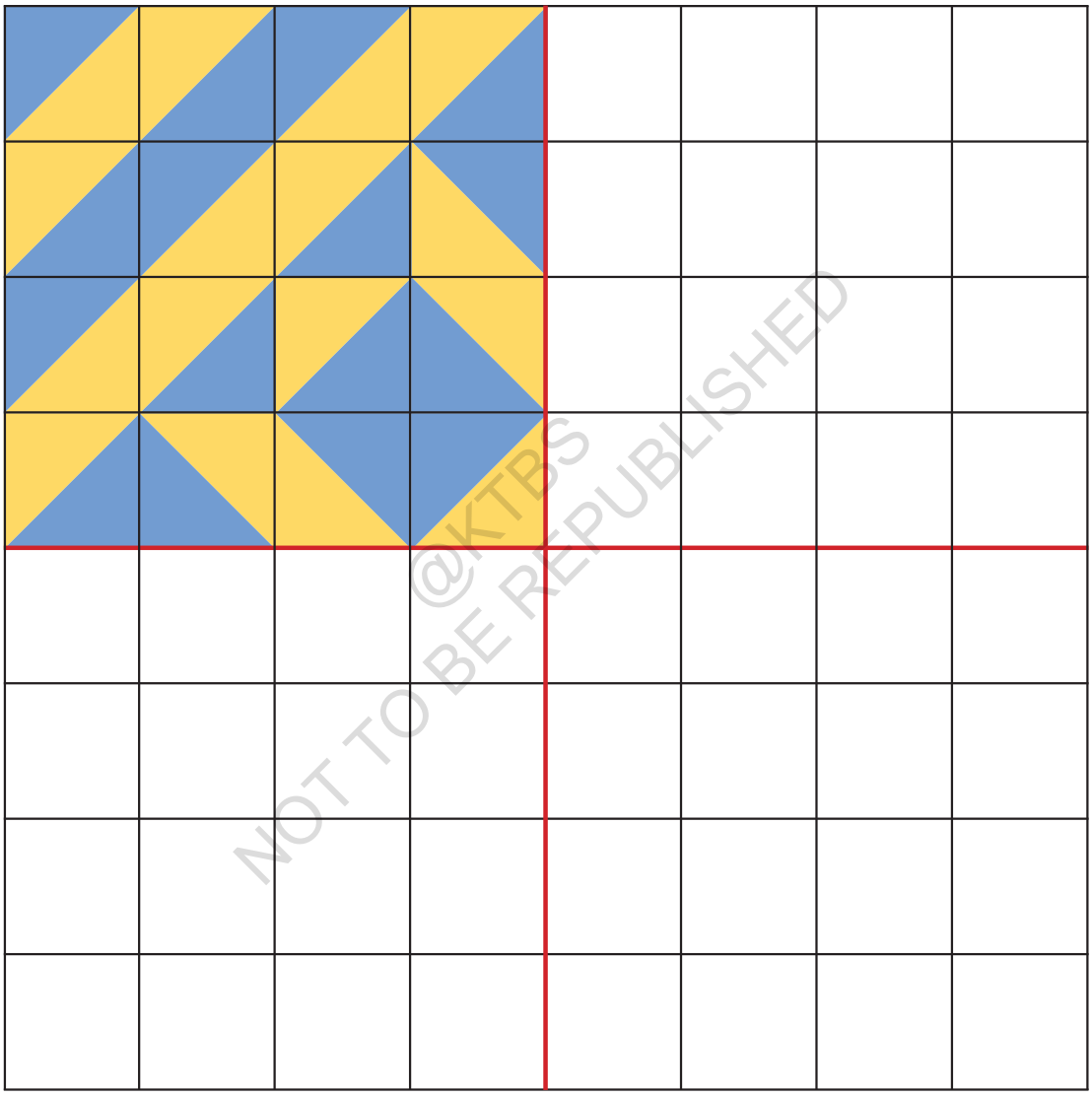


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Playing with Tiles

a. Use the colour tiles  given at the end of the book to complete the following figure so that it has exactly 2 lines of symmetry.
b. Use 16 such tiles to make figures that have exactly:
   1 line of symmetry
   2 lines of symmetry
c. Use these tiles in making creative symmetric designs.

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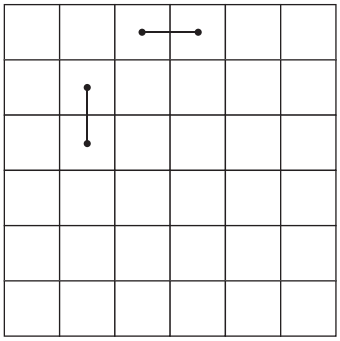


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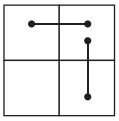
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Game

Draw a 6 by 6 grid. Two players take turns covering two adjacent squares by drawing a line. The line can be placed either way: horizontally or vertically. The lines cannot overlap. The game goes on till a player is not able to place any more lines. The player who is not able to place a line loses.



Not allowed



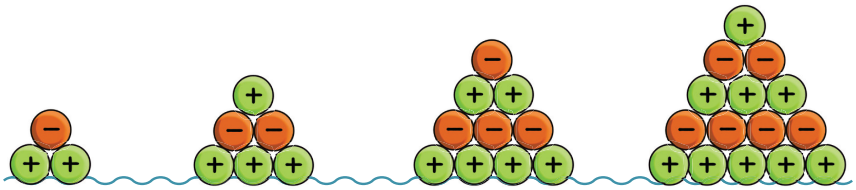
With what strategy can one play to win this game?

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SUMMARY

- When a figure is made up of parts that repeat in a definite pattern, we say that the figure has **symmetry**. We say that such a figure is **symmetrical**.
- A line that cuts a plane figure into two parts that exactly overlap when folded along that line is called a **line of symmetry** or **axis of symmetry** of the figure.
- A figure may have multiple lines of symmetry.
- Sometimes a figure looks exactly the same when it is rotated by an angle about a fixed point. Such an angle is called an **angle of symmetry** of the figure. A figure that has an angle of symmetry strictly between 0 and 360 degrees is said to have **rotational symmetry**. The point of the figure about which the rotation occurs is called the **centre of rotation**.
- A figure may have multiple angles of symmetry.
- Some figures may have a line of symmetry but no angle of symmetry, while others may have angles of symmetry but no lines of symmetry. Some figures may have both lines of symmetry as well as angles of symmetry.

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10 THE OTHER SIDE OF ZERO

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Integers

More and More Numbers!

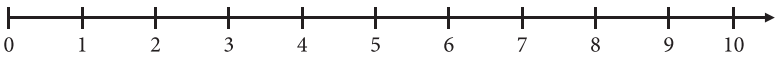
Recall that the very first numbers we learned about in the study of mathematics were the counting numbers 1, 2, 3, 4, ...

Then we learned that there are even more numbers! For example, there is the number 0 (zero), representing nothing, which comes before 1. The number 0 has a very important history in India and now in the world. For example, around the world we learn to write numbers in the Indian number system using the digits 0 to 9, allowing us to write numbers however large or however small using just these 10 digits.

We then learned about more numbers that exist between the numbers 0, 1, 2, 3, 4, ... , such as $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{13}{6}$. These are called *fractions*.

But are there still more numbers? Well, 0 is an additional number that we didn't know about earlier, and it comes before 1 and is less than 1. Are there perhaps more numbers that come before 0 and are less than 0?

Phrased another way, we have seen the number line:



However, this is actually only a number '*ray*', in the language we learned earlier in geometry; this ray starts at 0 and goes forever to the right. Do there exist numbers to the left of 0, so that this number ray can be completed to a true number line?

That is what we will investigate in this chapter!

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Can there be a number less than 0? Can you think of any ways to have less than 0 of something?

10.1 Bela's Building of Fun

Children flock to Bela's ice cream factory to see and taste her tasty ice cream. To make it even more fun for them, Bela purchased a multi-storied building and filled it with attractions. She named it *Bela's Building of Fun*.



But this was no ordinary building!

Observe that some of the floors in the 'Building of Fun' are below the ground. What are the shops that you find on these floors? What is there on the ground floor?

A lift is used to go up and down between the floors. It has two buttons: '+' to go up and '–' to go down. Can you spot the lift?

To go to the Art Centre from the 'Welcome Hall', you must press the '+' button twice.

We say that the button press is + + or +2.

To go down two floors, you must press the '–' button twice, which we write as – – or –2.

So if you press +1 (i.e., if you press the '+' button once), then you will go up one floor and if you press –1 (i.e., if you press the '–' button once), then you will go down 1 floor.



Lift button presses and numbers:
+++ is written as +3
– – – – is written as – 4

What do you press to go four floors up? What do you press to go three floors down?

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Numbering the floors in the building of fun

Entry to the 'Building of Fun' is at the ground floor level and is called the 'Welcome Hall'. Starting from the ground floor, you can reach the Food Court by pressing +1 and can reach the Art Centre by pressing +2. So, we can say that the Food Court is on Floor +1 and that the Art Centre is on Floor +2.

Starting from the ground floor, you must press –1 to reach the Toy Store. So, the Toy Store is on Floor –1 similarly starting from the ground floor, you must press –2 to reach the Video Games shop. So, the Video Games shop is on Floor –2.

The ground floor is called Floor 0. Can you see why?

Number all the floors in the Building of Fun.



Did you notice that +3 is the floor number of the Book Store, but it is also the number of floors you move when you press +3? Similarly, –3 is the floor number but it is also the number of floors you go down when you press –3, i.e., when you press – – – .

A number with a '+' sign in front is called a **positive number**. A number with a '–' sign in front is called a **negative number**.

In the 'Building of Fun', the floors are numbered using the ground floor, Floor 0, as a reference or starting point. The floors above the ground floor are numbered with positive numbers. To get to them from the ground floor, one must press the '+' button some number of times. The floors below the ground are numbered with negative numbers. To get to them from the ground floor, one must press the '–' button some number of times.

Zero is neither a positive nor a negative number. We do not put a '+' or '–' sign in front of it.

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Addition to keep track of movement

Start from the Food Court and press +2 in the lift. Where will you reach? _____

We can describe this using an expression:

Starting Floor + Movement = Target Floor.

The starting floor is +1 (Food Court) and the number of button presses is +2. Therefore, you reach the target floor **(+1) + (+2) = +3** (Book Store).

Figure it Out

1. You start from Floor +2 and press –3 in the lift. Where will you reach? Write an expression for this movement.
2. Evaluate these expressions (you may think of them as Starting Floor + Movement by referring to the Building of Fun).
   a. (+1) + (+4) = _____ b. (+4) + (+1) = _____
   c. (+4) + (–3) = _____ d. (–1) + (+2) = _____
   e. (–1) + (+1) = _____ f. 0 + (+2) = _____
   g. 0 + (–2) = _____
3. Starting from different floors, find the movements required to reach Floor –5. For example, if I start at Floor +2, I must press –7 to reach Floor –5. The expression is (+2) + (–7) = –5.
   Find more such starting positions and the movements needed to reach Floor –5 and write the expressions.

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Combining button presses is also addition

Gurmit was in the Toy Store and wanted to go down two floors. But by mistake he pressed the '+' button two times. He realised his mistake and quickly pressed the '–' button three times. How many floors below or above the Toy Store will Gurmit reach?

Gurmit will go one floor down. We can show the movement resulting from combining button presses as an expression: (+2) + (–3) = –1.

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Figure it out

Evaluate these expressions by thinking of them as the resulting movement of combining button presses:

a. $(+1)+(+4)=$ _____ b. $(+4)+(+1)=$ _____

c. $(+4)+(-3)+(-2)=$ _____ d. $(-1)+(+2)+(-3)=$ _____

Back to zero!



On the ground floor, Basant is in a great hurry and by mistake he presses +3. What can he do to cancel it and stay on the ground floor? He can cancel it by pressing –3. That is, (+3) + (–3) = 0.

We call –3 the inverse of +3. Similarly, the inverse of –3 is +3.

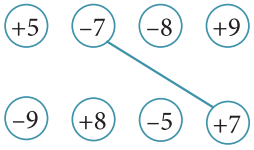
If Basant now presses +4 and then presses –4 in the lift, where will he reach?

Here is another way to think of the concept of inverse. If you are at Floor +4 and you press its inverse –4, then you are back to zero, the ground floor! If you are at Floor –2 and press its inverse +2, then you go to (–2) + (+2) = 0, again the ground floor!

Write the inverses of these numbers:

+4, –4, –3, 0, +2, –1.

Connect the inverses by drawing lines.



Comparing numbers using floors

Who is on the lowest floor?

1. Jay is in the Art Centre. So, he is on Floor +2.
2. Asin is in the Sports Centre. So, she is on Floor ___.
3. Binnu is in the Cinema Centre. So, she is on Floor ____.
4. Aman is in the Toys Store. So, he is on Floor ____.

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Floor +3 is lower than Floor +4. So, we write +3 < +4. We also write +4 > +3.

Should we write –3 < –4 or –4 < –3?

Floor –4 is lower than Floor –3. So, –4 < –3. It is also correct to write –3 > –4

Figure it Out

1. Compare the following numbers using the Building of Fun and fill in the boxes with < or >.

   a. –2 ☐ +5 b. –5 ☐ +4 c. –5 ☐ –3

   d. +6 ☐ –6 e. 0 ☐ –4 f. 0 ☐ +4

   Notice that all negative number floors are below Floor 0. So, all negative numbers are less than 0. All the positive number floors are above Floor 0. So, all positive numbers are greater than 0.

2. Imagine the Building of Fun with more floors. Compare the numbers and fill in the boxes with < or >:

   a. –10 ☐ –12 b. +17 ☐ –10

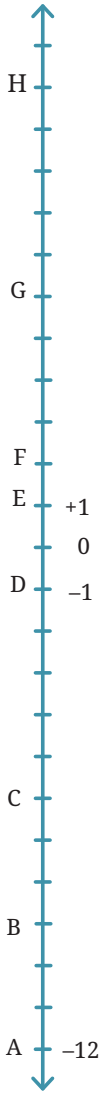
   c. 0 ☐ –20 d. +9 ☐ –9

   e. –25 ☐ –7 f. +15 ☐ –17

3. If Floor A = –12, Floor D = –1 and Floor E = +1 in the building shown on the right as a line, find the numbers of Floors B, C, F, G, and H.

4. Mark the following floors of the building shown on the right.

   a. –7 b. –4 c. +3 d. –10



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Subtraction to find which button to press

In earlier classes, we understood the meaning of subtraction as 'take away'. For example, there are 10 books on the shelf. I take away 4 books. How many are left on the shelf?

We can express the answer using subtraction: 10 – 4 = 6 or 'Ten take away four is six.'

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You may also be familiar with another meaning of subtraction which is related to comparison or making quantities equal. For example, consider this situation: I have ₹10 with me and my sister has ₹6.

Now, I can ask the question: 'How much more money should my sister get in order to have the same amount as me?'

We can write this in two ways: 6 + ? = 10 Or 10 – 6 = ?

Here, we see the connection between 'finding the missing number to be added' and subtraction.

For subtraction of positive and negative numbers, we will use this meaning of subtraction as 'making equal' or 'finding the missing number to be added'.

Evaluate 15 – 5, 100 – 10 and 74 – 34 from this perspective.

Teacher's Note

In general, when there are two unequal quantities, subtraction can indicate the change needed to make the quantities equal. Subtraction shows how much the starting quantity should change in order to become the target quantity. In the context of different floor levels, what is the change required to reach the Target Floor from the Starting Floor? Notice that the change needed may be positive (for an increase) or negative (for a decrease).

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Your starting floor is the Art Centre and your target floor is the Sports Centre. What should be your button press?

You need to go three floors up, so you should press +3. We can write this as an expression using subtraction:

Target Floor – Starting Floor = Movement needed.

In the above example, the starting floor is +2 (Art Centre) and the target floor is +5. The button press to get to +5 from +2 is +3. Therefore,

(+5) – (+2) = +3

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Explanation

Recall the connection between addition and subtraction. For 3 + ? = 5, we can find the missing number using subtraction: 5 – 3 = 2. That is, subtraction is the same as finding the missing number to be added.

We know that —

Starting Floor + Movement needed = Target Floor

If the movement needed is to be found, then,

Starting Floor + ? = Target Floor

So,

Target Floor – Starting Floor = ? = Movement needed

More examples:

a. If the Target Floor is –1 and Starting Floor is –2, what button should you press?
You need to go one floor up, so, you should press +1.
Expression: **(– 1) – (– 2) = (+1)**

b. If the Target Floor is –1 and Starting floor is +3, what button should you press?
You need to go four floors down, so, you should press –4.
Expression: **(– 1) – (+ 3) = (– 4)**

c. If the Target Floor is +2 and Starting Floor is –2, what button should you press?
You need to go four floors up, so, you should press +4.
Expression: **(+ 2) – (– 2) = (+ 4)**

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Figure it Out

Complete these expressions. You may think of them as finding the movement needed to reach the Target Floor from the Starting Floor.

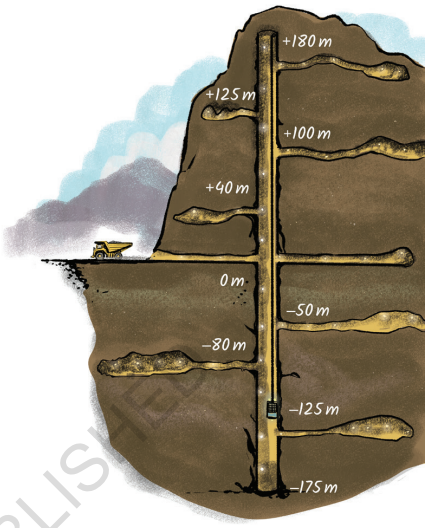
a. (+1) – (+4) = _____
b. (0) – (+2) = _____
c. (+4) – (+1) = _____
d. (0) – (–2) = _____
e. (+4) – (–3) = _____
f. (–4) – (–3) = _____
g. (–1) – (+2) = _____
h. (–2) – (–2) = _____
i. (–1) – (+1) = _____
j. (+3) – (–3) = _____

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Adding and subtracting larger numbers

The picture shows a mine, a place where minerals are extracted by digging into the rock. The truck is at the ground level, but the minerals are present both above and below the ground level. There is a fast moving lift which moves up and down in a mineshaft carrying people and ore.



Some of the levels are marked in the picture. The ground level is marked 0. Levels above the ground are marked by positive numbers and levels below the ground are marked by negative numbers. The number indicates how many metres above or below the ground level it is.

In the mine, just like in the Building of Fun:

Starting Level + Movement = Target Level

For example,
$(+40) + (+60) = +100$ $(-90) + (-55) = -145$

Target Level – Starting Level = Movement needed

For example,
$(+40) - (-50) = +90$ $(-90) - (+40) = -130$

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How many negative numbers are there?

Bela's Building of Fun had only six floors above and five floors below. That is numbers –5 to +6. In the mine above, we have numbers from –200 to +180. But we can imagine larger buildings or mineshafts. Just as positive numbers +1, +2, +3, ... keep going up without an end, similarly, negative numbers –1, –2, –3, ... keep going down. Positive and negative numbers, with zero, are called **integers**. They go both ways from 0: ... –4, –3, –2, –1, 0, 1, 2, 3, 4, ...

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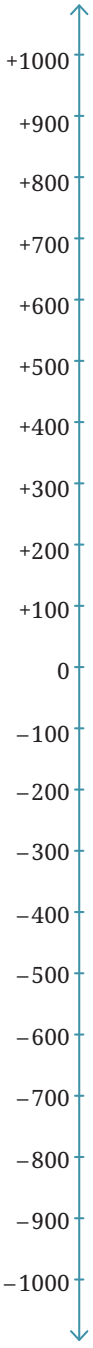


Figure it Out

Complete these expressions.

a. (+40) + _____ = +200
b. (+40) + _____ = −200
c. (−50) + _____ = +200
d. (−50) + _____ = −200
e. (−200) − (−40) = _____
f. (+200) − (+40) = _____
g. (−200) − (+40) = _____

Check your answers by thinking about the movement in the mineshaft.

Adding, subtracting, and comparing any numbers

To add and subtract even larger integers, we can imagine even larger lifts! In fact, we can imagine a lift that can extend forever upwards and forever downwards, starting from Level 0. There does not even have to be any building or mine around — just an 'infinite lift'!

We can use this imagination to add and subtract any integers we like.

For example, suppose we want to carry out the subtraction +2000 − (−200). We can imagine a lift with 2000 levels above the ground and 200 below the ground. Recall that,

Target Level − Starting Level = Movement needed

To go from the Starting Floor −200 to the Target Floor +2000, we must press +2200 (+200 to get to zero, and then +2000 more after that to get to +2200). Therefore, (+2000) − (−200) = +2200.

Notice that (+2000) + (+200) is also +2200.

Try evaluating the following expressions by similarly drawing or imagining a suitable lift:

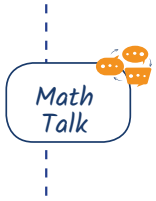
a. −125 + (−30)
b. +105 − (−55)
c. +105 + (+55)
d. +80 − (−150)
e. +80 + (+150)
f. −99 − (−200)
g. −99 + (+200)
h. +1500 − (−1500)

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In the above example, we saw that + 2000 – (– 200) = + 2000 + (+ 200) = + 2200. In other words, subtracting a negative number is the same as adding the corresponding positive number. That is, we can replace subtraction of a negative number by addition of a positive number!

In the other exercises that you did above, did you notice that subtracting a negative number was the same as adding the corresponding positive number?



Take a look at the 'infinite lift' above. Does it remind you of a number line? In what ways?

Back to the number line

The 'infinite lift' we saw above looked very much like a number line, didn't it? In fact, if we rotate it by 90°, it basically becomes a number line. It also tells us how to complete the number *ray* to a number line, answering the question that we had asked at the beginning of the chapter. To the left of 0 are the negative numbers –1, –2, –3, ...

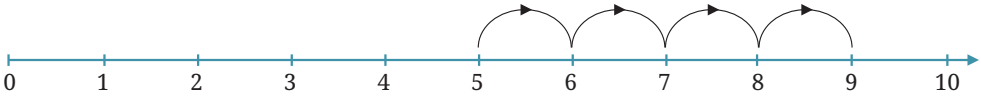
Usually we drop the '+' sign on positive numbers and simply write them as 1, 2, 3, ...



Instead of traveling along the number line using a lift, we can simply imagine walking on it. To the right is the positive (forward) direction, and to the left is the negative (backward) direction.

Smaller numbers are now to the left of bigger numbers and bigger numbers are to the right of smaller numbers. So, 2 < 5; –3 < 2; and –5 < –3.

If, from 5 you wish to go over to 9, how far must you travel along the number line?



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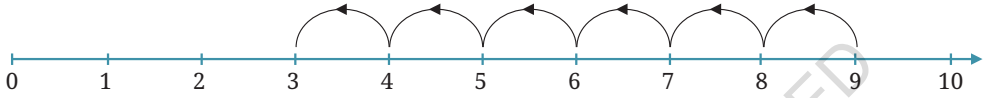
You must travel 4 steps. That is why 5 + 4 = 9.

(Remember: **Starting Number + Movement = Target Number**)

The corresponding subtraction statement is 9 – 5 = 4.

(Remember: **Target Number – Starting Number = Movement needed**)

Now, from 9, if you wish to go to 3, how much must you travel along the number line?



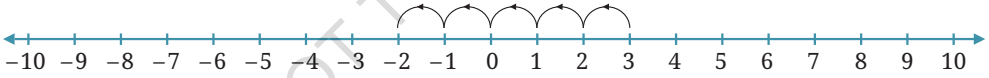
You must move 6 steps backward, i.e., you must move –6. Hence, we write 9 + (–6) = 3.

(Remember again : **Starting Number + Movement = Target Number**)

The corresponding subtraction statement is 3 – 9 = – 6.

(Remember again: **Target Number – Starting Number = Movement needed**)

Now, from 3, if you wish to go to –2, how far must you travel?



You must travel –5 steps, i.e., 5 steps backward. Thus, 3 + (–5) = –2. The corresponding subtraction statement is: –2 – 3 = –5.

Figure it Out



1. Mark 3 positive numbers and 3 negative numbers on the number line above.
2. Write down the above 3 marked negative numbers in the following boxes: ▢ ▢ ▢

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3. Is 2 > –3? Why? Is –2 < 3? Why?
4. What are a. –5 + 0 b. 7 + (–7) c. –10 + 20 d. 10 – 20 e. 7 – (–7) f. –8 – (–10)?

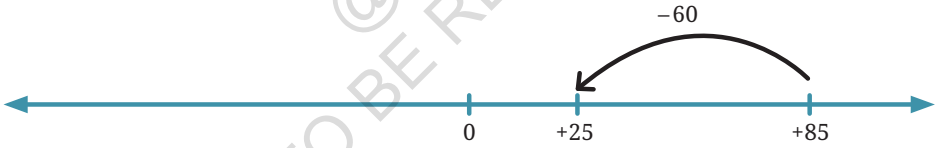
Using the unmarked number line to add and subtract

Just as you can do additions, subtractions and comparisons with small numbers using the number line above, you can also do them with large numbers by imagining an 'infinite number line' or drawing an 'unmarked number line' as follows:



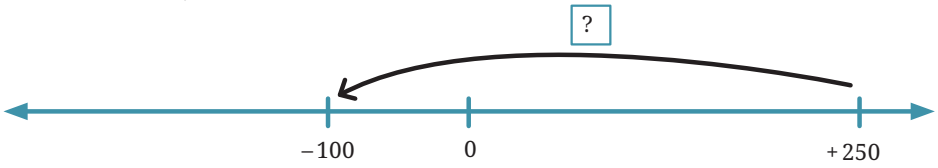
This line shows only the position of zero. Other numbers are not marked. It can be convenient to use this **u**nmarked **n**umber **l**ine to add and subtract integers. You can show, or simply imagine, the scale of the number line and the positions of numbers on it.

For example, this unmarked number line (UNL) shows the addition problem: 85 + (–60) = ?



We then can visualise that 85 + (–60) = 25

The following UNL shows a subtraction problem which can also be written as a missing addend problem: (–100) – (+250) = ? or 250 + ? = –100.



We can then visualise that ? = –350 in this problem.

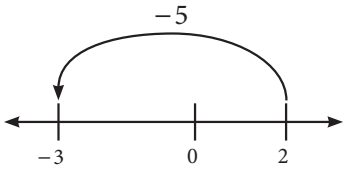
In this way, you can carry out addition and subtraction problems, with positive and negative numbers, on paper or in your head using an unmarked number line.

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Use unmarked number lines to evaluate these expressions:

a. $-125 + (-30) =$ _____
b. $+105 - (-55) =$ _____
c. $+80 - (-150) =$ _____
d. $-99 - (-200) =$ _____



Converting subtraction to addition and addition to subtraction

Recall that **Target Floor – Starting Floor = Movement needed**

or

Target Floor = Starting Floor + Movement needed

If we start at 2 and wish to go to –3, what is the movement needed?

First method: Looking at the number line, we see we need to move –5 (i.e., 5 in the backward direction). Therefore, **$-3 - 2 = -5$**. The movement needed is –5.

Second method: Break the journey from 2 to –3 into two parts.

a. From 2 to 0, the movement is $0 - 2 = -2$.
b. From 0 to –3, the movement is $-3 - 0 = -3$.

The total movement is the sum of the two movements: **$-3 + (-2) = -5$**.

Look at the two coloured expressions. There is no subtraction in the second one!

In this way, we can always convert subtraction to addition. **The number that is being subtracted can be replaced by its inverse and then added instead.**

Similarly, a number that is being added can be replaced by its inverse and then subtracted. In this way, we can also always convert addition to subtraction.

Examples:

a. $(+7) - (+5) = (+7) + (-5)$
b. $(-3) - (+8) = (-3) + (-8)$
c. $(+8) - (-2) = (+8) + (+2)$
d. $(+6) - (-9) = (+6) + (+9)$

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10.2 The Token Model

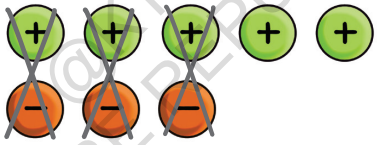
Using tokens for addition

In Bela's Building of Fun, the lift attendant is bored. To entertain himself, he keeps a box containing lots of positive (red) and negative (black) tokens. Each time he presses the '+' button, he takes a positive token from the box and puts it in his pocket. Similarly, each time he presses the '–' button, he takes a negative token and puts it in his pocket.

He starts on the ground floor (Floor 0) with an empty pocket. After one hour, he checks his pocket and finds 5 positive and 3 negative tokens. On which floor is he now?

He must have pressed '+' five times and '–' 3 times and $(+5)+(-3)=+2$. So, he is at Floor +2 now.

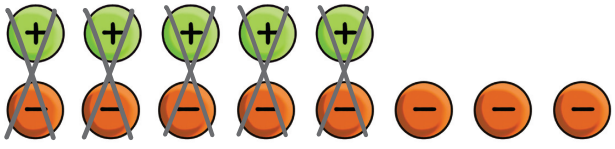
Here is another way to do the calculation.



A positive token and a negative token cancel each other, because the value of this pair of tokens together is zero. These two tokens in his pocket meant that he pressed '+' once and '–' once, respectively, and these cancel each other. We say that a positive and a negative token make a 'zero pair'. When you remove all the zero pairs, you are left with two positive tokens, so (+5) + (–3) = +2.

We can perform any such addition using tokens!

Example: Add +5 and –8.



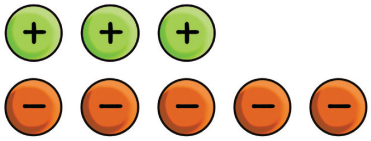
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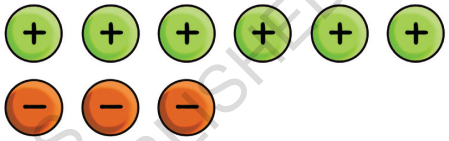
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From the picture, we see that we can remove five zero pairs, and we are then left with –3. Therefore (+5) + (–8) = –3.

Figure it Out

1. Complete the additions using tokens.
   a. (+6) + (+4) b. (–3) + (–2)
   c. (+5) + (–7) d. (–2) + (+6)
2. Cancel the zero pairs in the following two sets of tokens. On what floor is the lift attendant in each case? What is the corresponding addition statement in each case?

a. 

b. 

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Using tokens for subtraction

We have seen how to perform addition of integers with positive tokens and negative tokens. We can also perform subtraction using tokens!

Example: Let us subtract:
(+5) – (+4).



(+5) – (+4) = +1

This is easy to do. From 5 positives take away 4 positives to see the result.

Example: Let us subtract:
(–7) – (–5).



(–7) – (–5) = –2

Is (–7) – (–5) the same as (–7) + (+5)?

Example: Let us subtract: (+5) – (+6).
Put down 5 positives.

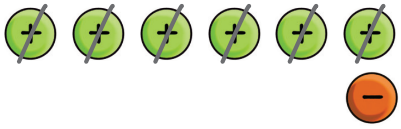


But there are not enough tokens to take out 6 positives!

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To get around this issue, we can put out an extra zero pair (a positive and a negative), knowing that this does not change the value of the set of tokens.

Now, we can take out 6 positives! See what is left:



We conclude that (+5) – (+6) = –1.

Figure it Out

1. Evaluate the following differences using tokens. Check that you get the same result as with other methods you now know:
   a. (+10)–(+7) b. (–8)–(–4) c. (–9)–(–4)
   d. (+9)–(+12) e. (–5)–(–7) f. (–2)–(–6)
2. Complete the subtractions:
   a. (–5)–(–7) b. (+10)–(+13) c. (–7)–(–9)
   d. (+3)–(+8) e. (–2)–(–7) f. (+3)–(+15)

Example: +4 – (–6).

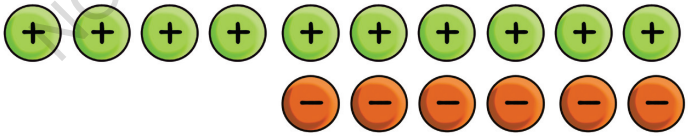
Start with 4 positives.



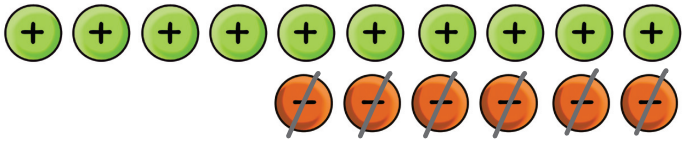
We have to take out 6 negatives from these. But there are not enough negatives.

This is not a problem. We add some zero pairs as this does not change the value of the set of tokens.

But how many zero pairs? We have to take away 6 negatives so we put down 6 zero pairs:



Now we can take away 6 negatives:



Therefore, +4–(–6) = +10.

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Figure it Out

1. Try to subtract: $-3-(+5)$.
   How many zero pairs will you have to put in? What is the result?
2. Evaluate the following using tokens.
   a. $(-3)-(+10)$ b. $(+8)-(-7)$ c. $(-5)-(+9)$
   d. $(-9)-(+10)$ e. $(+6)-(-4)$ f. $(-2)-(+7)$

10.3 Integers in Other Places

Credits and debits

Suppose you open a bank account at your local bank with the ₹100 that you had been saving over the last month. Your bank balance therefore, starts at ₹100.

Then you make ₹60 at your job the next day and you deposit it in your account. This is shown in your bank passbook as a 'credit'.

Your new bank balance is _____.

The next day you pay your electric bill of ₹30 using your bank account. This is shown in your bank passbook as a 'debit'.

Your bank balance is now _____.

The next day you make a major purchase for your business of ₹150. Again this is shown as a debit.

What is your bank balance now? _____

Is this possible?

Yes, some banks do allow your account balance to become negative, temporarily! Some banks also charge you an additional amount if your balance becomes negative, in the form of 'interest' or a 'fee'.

Your strategic large purchase the previous day allows you to make, ₹200 at your business the next day.

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What is your balance now? _____

You can think of 'credits' as positive numbers and 'debits' as negative numbers. The total of all your credits (positive numbers) and debits (negative numbers) is your total bank account balance. This can be positive or negative!

In general, it is better to try to keep a positive balance in your bank account!

Figure it Out

1. Suppose you start with ₹0 in your bank account, and then you have credits of ₹30, ₹40, and ₹50, and debits of ₹40, ₹50, and ₹60. What is your bank account balance now?
2. Suppose you start with ₹0 in your bank account, and then you have debits of ₹1, 2, 4, 8, 16, 32, 64, and 128, and then a single credit of ₹256. What is your bank account balance now?
3. Why is it generally better to try and maintain a positive balance in your bank account? What are circumstances under which it may be worthwhile to temporarily have a negative balance?

As you can see, positive and negative numbers along with zero are extremely useful in the world of banking and accounting.

Geographical cross sections

We measure the height of geographical features like mountains, plateaus, and deserts from 'sea level'. The height at sea level is 0m. Heights above sea level are represented using positive numbers and heights below sea level are represented using negative numbers.

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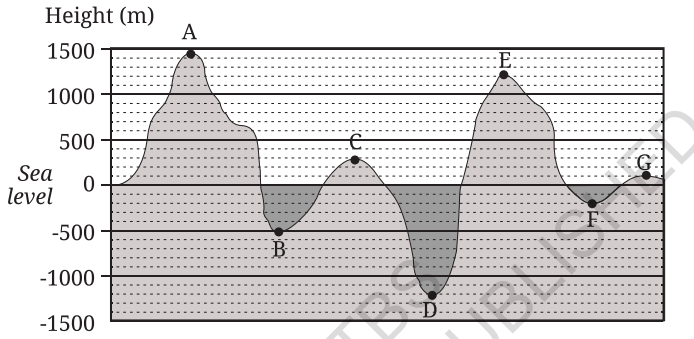
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Figure it Out

1. Looking at the geographical cross section, fill in the respective heights:

   a. ☐ b. ☐ c. ☐ d. ☐

   e. ☐ f. ☐ g. ☐



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Teacher's Note

Ask what a geographical cross section is by showing the figure in this page. It is like imagining a vertical slice taken out at some location on the Earth. This is what would be seen from a side view. Discuss the notion of 'sea level' for measuring heights and depths in geography.

2. Which is the highest point in this geographical cross section? Which is the lowest point?
3. Can you write the points A, B, ..., G in a sequence of decreasing order of heights? Can you write the points in a sequence of increasing order of heights?
4. What is the highest point above sea level on Earth? What is its height?
5. What is the lowest point with respect to sea level on land or on the ocean floor? What is its height? (This height should be negative).

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Temperature

During summer time you would have heard in the news that there is a 'heat wave'. What do you think will be the temperature during the summer when you feel very hot? In the winter we have cooler or colder temperatures.

What has been the maximum temperature during the summer and the minimum temperature during the winter last year in your area? Find out.

When we measure temperature, we use Celsius as the unit of measure (°C). The thermometers below are showing 40°C and 15°C temperatures.

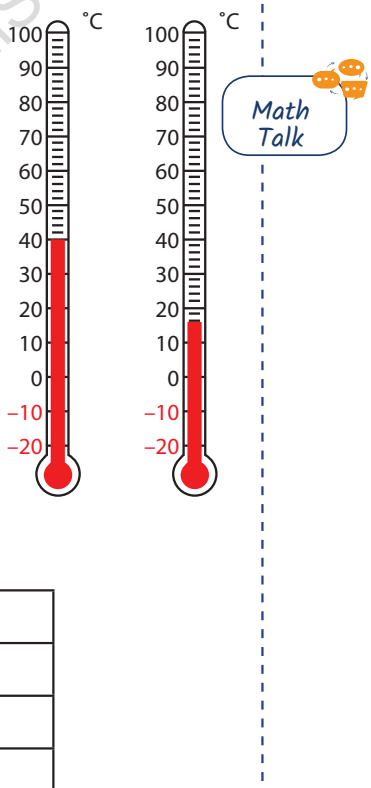


Figure it Out

1. Do you know that there are some places in India where temperatures can go below 0°C? Find out the places in India where temperatures sometimes go below 0°C. What is common among these places? Why does it become colder there and not in other places?
2. Leh in Ladakh gets very cold during the winter. The following is a table of temperature readings taken during different times of the day and night in Leh on a day in November. Match the temperature with the appropriate time of the day and night.

Temperature
14°C
8°C
–2°C
–4°C

Time
02:00 a.m.
11:00 p.m.
02:00 p.m.
11:00 a.m.

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Teacher's Note

Talk about thermometers and how they are used to measure temperature. Bring a laboratory thermometer to the class and measure the temperature of hot water and cold water. Point out to children that there are markings in the thermometer that are below 0°C. Have a discussion on what 0°C indicates, namely, the freezing point of water.

10.4 Explorations with Integers

A hollow integer grid

4	−1	−3
−3		1
−1	−1	2

5	−3	−5
0		−5
−8	−2	7

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There is something special about the numbers in these two grids. Let us explore what that is.

Top row:	$4 + (-1) + (-3) = 0$	$5 + (-3) + (-5) =$ ____
Bottom row:	$(-1) + (-1) + 2 = 0$	$(-8) + (-2) + 7 =$ ____
Left column:	$4 + (-3) + (-1) = 0$	$5 + 0 + (-8) =$ ____
Right column:	$(-3) + 1 + 2 = 0$	$(-5) + (-5) + 7 =$ ____

In each grid, the numbers in each of the two rows (the top row and the bottom row) and the numbers in each of the two columns (the leftmost column and the rightmost column) add up to give the same number. We shall call this sum as the 'border sum'. The border sum of the first grid is '0'.

Figure it Out

1. Do the calculations for the second grid above and find the border sum.

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2. Complete the grids to make the required border sum:

–10		
		–5
9		

Border sum is +4

6	8	
		–5
	–2	

Border sum is –2

7		
		–5

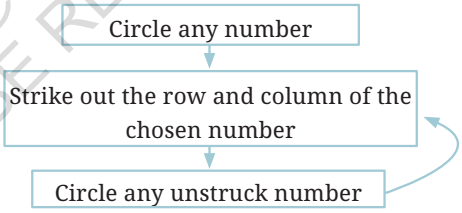
Border sum is –4

3. For the last grid above, find more than one way of filling the numbers to get border sum –4.
4. Which other grids can be filled in multiple ways? What could be the reason?
5. Make a border integer square puzzle and challenge your classmates.

An amazing grid of numbers!

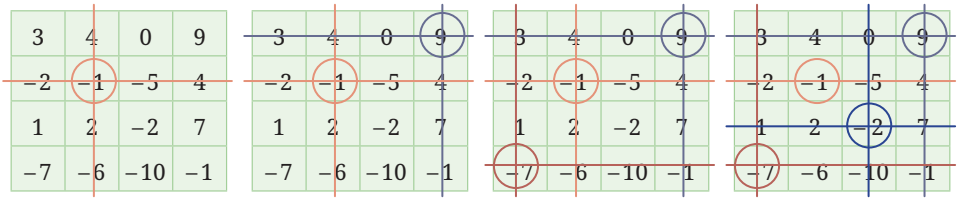
Below is a grid having some numbers. Follow the steps as shown until no number is left.

3	4	0	9
–2	–1	–5	4
1	2	–2	7
–7	–6	–10	–1



When there are no more unstruck numbers, STOP. Add the circled numbers.

In the example below, the circled numbers are –1, 9, –7, –2. If you add them, you get –1.



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Figure it Out

1. Try afresh, choose different numbers this time. What sum did you get? Was it different from the first time? Try a few more times!
2. Play the same game with the grids below. What answer did you get?

7	10	13	16
−2	1	4	7
−11	−8	−5	−2
−20	−7	−14	−11

−11	−10	−9	−8
−7	−6	−5	−4
−3	−2	−1	0
1	2	3	4

3. What could be so special about these grids? Is the magic in the numbers or the way they are arranged or both? Can you make more such grids?



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Figure it Out

1. Write all the integers between the given pairs, in increasing order.
   a. 0 and −7 b. −4 and 4
   c. −8 and −15 d. −30 and −23
2. Give three numbers such that their sum is −8.
3. There are two dice whose faces have these numbers: −1, 2, −3, 4, −5, 6. The smallest possible sum upon rolling these dice is −10 = (−5) + (−5) and the largest possible sum is 12 = (6) + (6). Some numbers between (−10) and (+12) are not possible to get by adding numbers on these two dice. Find those numbers.
4. Solve these:

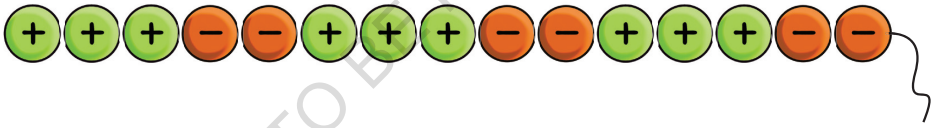
8 − 13	(−8) − (13)	(−13) − (−8)	(−13) + (−8)
8 + (−13)	(−8) − (−13)	(13) − 8	13 − (−8)

5. Find the years below.
   a. From the present year, which year was it 150 years ago? _____
   b. From the present year, which year was it 2200 years ago? _____
   Hint: Recall that there was no year 0.

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c. What will be the year 320 years after 680 BCE? _____

6. Complete the following sequences:
   a. (–40), (–34), (–28), (–22), _____, _____, _____
   b. 3, 4, 2, 5, 1, 6, 0, 7, _____, _____, _____
   c. _____, _____, 12, 6, 1, (–3), (–6), _____, _____, _____
7. Here are six integer cards: (+1), (+7), (+18), (–5), (–2), (–9).
   You can pick any of these and make an expression using addition(s) and subtraction(s).
   Here is an expression: (+18) + (+1) – (+7) – (–2) which gives a value (+14). Now, pick cards and make an expression such that its value is closer to (– 30).
8. The sum of two positive integers is always positive but a (positive integer) – (positive integer) can be positive or negative. What about
   a. (positive) – (negative)
   b. (positive) + (negative)
   c. (negative) + (negative)
   d. (negative) – (negative)
   e. (negative) – (positive)
   f. (negative) + (positive)
9. This string has a total of 100 tokens arranged in a particular pattern. What is the value of the string?



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10.5 A Pinch of History

Like general fractions, general integers (including zero and the negative numbers) were first conceived of and used in Asia, thousands of years ago, before they eventually spread across the world in more modern times.

The first known instances of the use of negative numbers occurred in the context of accounting. In one of China's most important mathematical works, *The Nine Chapters on Mathematical Art (Jiuzhang Suanshu)*—which was completed by the first or second century CE—positive and negative numbers were represented using red and black rods, much like the way we represented them using red and black tokens!

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There was a strong culture of accountancy also in India in ancient times. The concept of credit and debit was written about extensively by Kautilya in his *Arthaśhāstra* (c. 300 BCE), including the recognition that an account balance could be negative. The explicit use of negative numbers in the context of accounting is seen in a number of ancient Indian works, including in the *Bakśhālī* manuscript from around the year 300 CE, where a negative number was written using a special symbol placed after the number (rather than before the number as we do today).

The first general treatment of positive numbers, negative numbers, and *zero*—all on an equal footing as equally-valid numbers on which one can perform the basic operations of addition, subtraction, multiplication, and even division—was given by Brahmagupta in his *Brāhma-sphuṭa-siddhānta* in the year 628 CE. Brahmagupta gave clear and explicit rules for operations on all numbers—positive, negative, and zero—that essentially formed the modern way of understanding these numbers that we still use today!

Some of Brahmagupta's key rules for addition and subtraction of positive numbers, negative numbers and zero are given below:

Brahmagupta's Rules for Addition (Brāhma-sphuṭa-siddhānta 18.30, 628 CE)

1. The sum of two positives is positive (e.g., 2 + 3 = 5).
2. The sum of two negatives is negative. To add two negatives, add the numbers (without the signs), and then place a minus sign to obtain the result (e.g., (–2) + (–3) = –5).
3. To add a positive number and a negative number, subtract the smaller number (without the sign) from the greater number (without the sign), and place the sign of the greater number to obtain the result (e.g., – 5 + 3 = –2, 2 + (–3) = –1 and –3 + 5 = 2).
4. The sum of a number and its inverse is zero (e.g., 2 + (–2) = 0).
5. The sum of any number and zero is the same number (e.g., –2 + 0 = –2 and 0 + 0 = 0).

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Brahmagupta's Rules for Subtraction (Brāhma-sphuṭa-siddhānta 18.31-18.32)

1. If a smaller positive is subtracted from a larger positive, the result is positive (e.g., 3 – 2 = 1).
2. If a larger positive is subtracted from a smaller positive, the result is negative (e.g., 2–3 = –1).
3. Subtracting a negative number is the same as adding the corresponding positive number (e.g., 2 – (–3) = 2 + 3).
4. Subtracting a number from itself gives zero (e.g., 2 – 2 = 0 and –2 – (–2) = 0).
5. Subtracting zero from a number gives the same number (e.g., –2 – 0 = –2 and 0 – 0 = 0). Subtracting a number from zero gives the number's inverse (e.g., 0 – (–2) = 2).

Once you understand Brahmagupta's rules, you can do addition and subtraction with any numbers whatsoever — positive, negative, and zero!

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Figure it Out

1. Can you explain each of Brahmagupta's rules in terms of Bela's Building of Fun, or in terms of a number line?
2. Give your own examples of each rule.

Brahmagupta was the first to describe zero as a number on an equal footing with positive numbers as well as with negative numbers, and the first to give explicit rules for performing arithmetic operations on all such numbers, positive, negative, and zero—forming what is now called a *ring*. It would change the way the world does mathematics.

However, it took many centuries for the rest of the world to adopt zero and negative numbers as numbers. These numbers were transmitted to, accepted by, and further studied by the Arab world by the 9th century, before making their way to Europe by the 13th century.

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Surprisingly, negative numbers were still not accepted by many European mathematicians even in the 18th century. Lazare Carnot, a French mathematician in the 18th century, called negative numbers 'absurd'. But over time, zero as well as negative numbers proved to be indispensable in global mathematics and science, and are now considered to be critical numbers on an equal footing with and as important as positive numbers—just as Brahmagupta had recommended and explicitly described way back in the year 628 CE! This abstraction of arithmetic rules on all numbers paved the way for the modern development of algebra, which we will learn about in future classes.

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SUMMARY

- There are numbers that are less than zero. They are written with a '–' sign in front of them (e.g., –2), and are called **negative numbers**. They lie to the left of zero on the number line.
- The numbers ..., –4, –3, –2, –1, 0, 1, 2, 3, 4, ... are called **integers**. The numbers 1, 2, 3, 4, ... are called **positive integers** and the numbers ..., –4, –3, –2, –1 are called **negative integers**. Zero (0) is neither positive nor negative.
- Every given number has another number associated to it which when added to the given number gives zero. This is called the **additive inverse** of the number. For example, the additive inverse of 7 is –7 and the additive inverse of –543 is 543.
- Addition can be interpreted as **Starting Position** + **Movement** = **Target Position**.
- Addition can also be interpreted as the combination of movements or increases/decreases: **Movement 1** + **Movement 2** = **Total Movement**.
- Subtraction can be interpreted as **Target Position** – **Starting Position** = **Movement**.

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- In general, we can add two numbers by following Brahmagupta's Rules for Addition:
  a. If both numbers are positive, add the numbers and the result is a positive number (e.g., 2 + 3 = 5).
  b. If both numbers are negative, add the numbers (without the signs), and then place a minus sign to obtain the result (–2 + (–3) = –5).
  c. If one number is positive and the other is negative, subtract the smaller number (without the sign) from the greater number (without the sign), and place the sign of the greater number to obtain the result (e.g., –5 + 3 = –2).
  d. A number plus its additive inverse is zero (e.g., 2 + (–2) = 0).
  e. A number plus zero gives back the same number (e.g., –2 + 0 = –2).
- We can subtract two integers by converting the problem into an addition problem and then following the rules of addition. Subtraction of an integer is the same as the addition of its additive inverse.
- Integers can be compared: ... –3 < –2 < –1 < 0 < +1 < +2 < +3 < ... Smaller numbers are to the left of larger numbers on the number line.
- We can give meaning to positive and negative numbers by interpreting them as credits and debits. We can also interpret positive numbers as distances above a reference point like the ground level. Similarly, negative numbers can be interpreted as distances below the ground level. When measuring temperatures in degrees Celsius, positive temperatures are those above the freezing point of water, and negative temperatures are those below the freezing point of water.

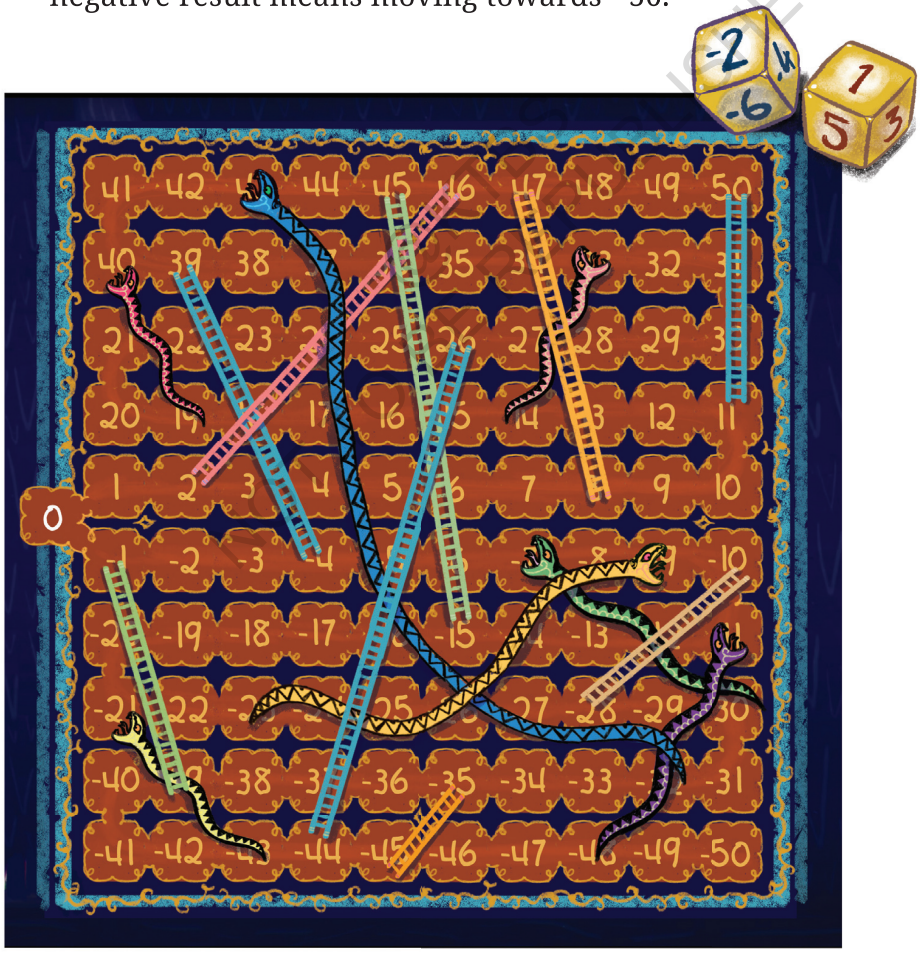
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Integers: Snakes and Ladders

Rules

- This is a two player game. Each player has 1 pawn. Both players start at 0. Players can reach either –50 or +50 to win but need not decide or fix this before or during play.
- Each player rolls two dice at a time. One dice has numbers from +1 to +6 and the other dice has numbers from –1 to –6.
- After each roll of the two dice, the player can add or subtract them in any order and then move the steps that indicate the result. A positive result means moving towards +50 and a negative result means moving towards –50.



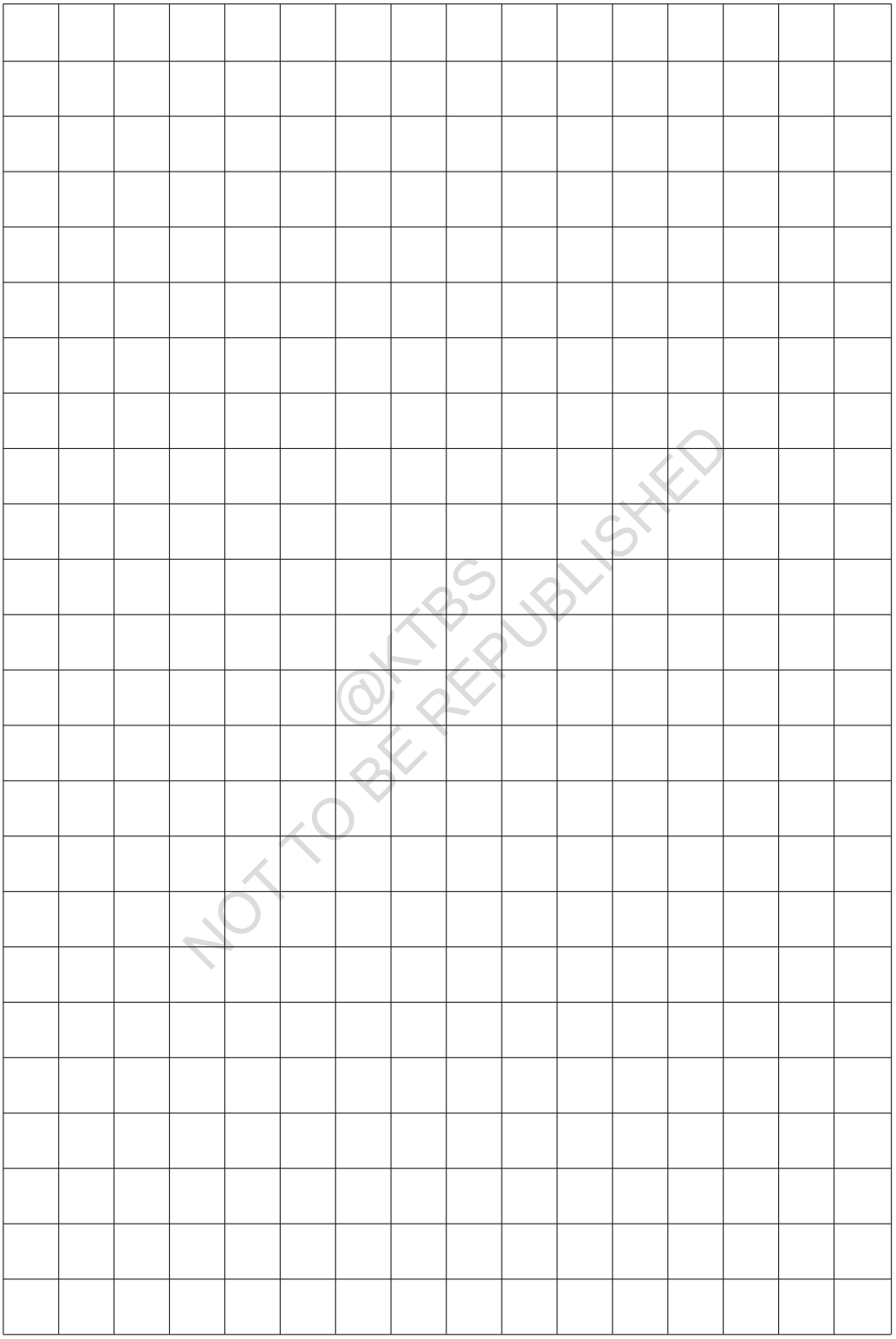
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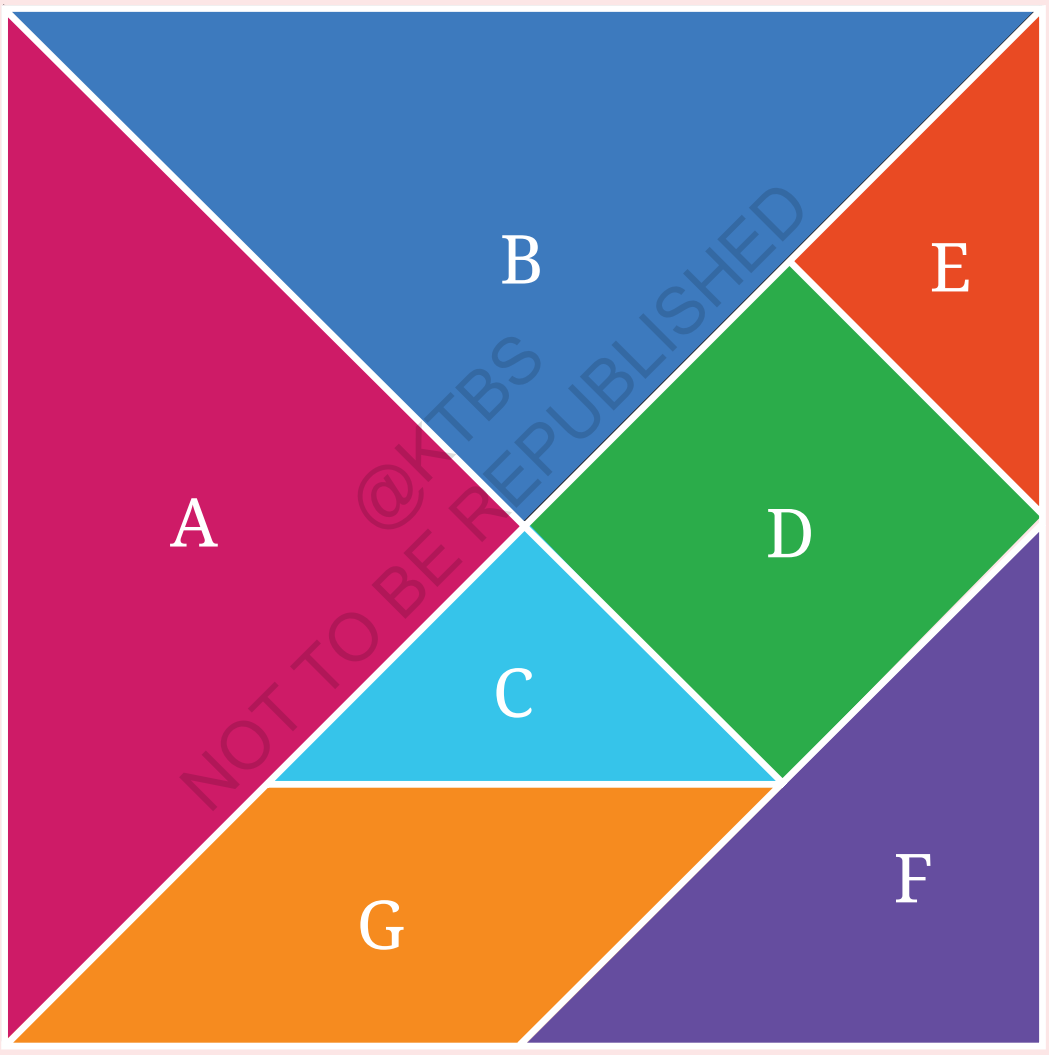
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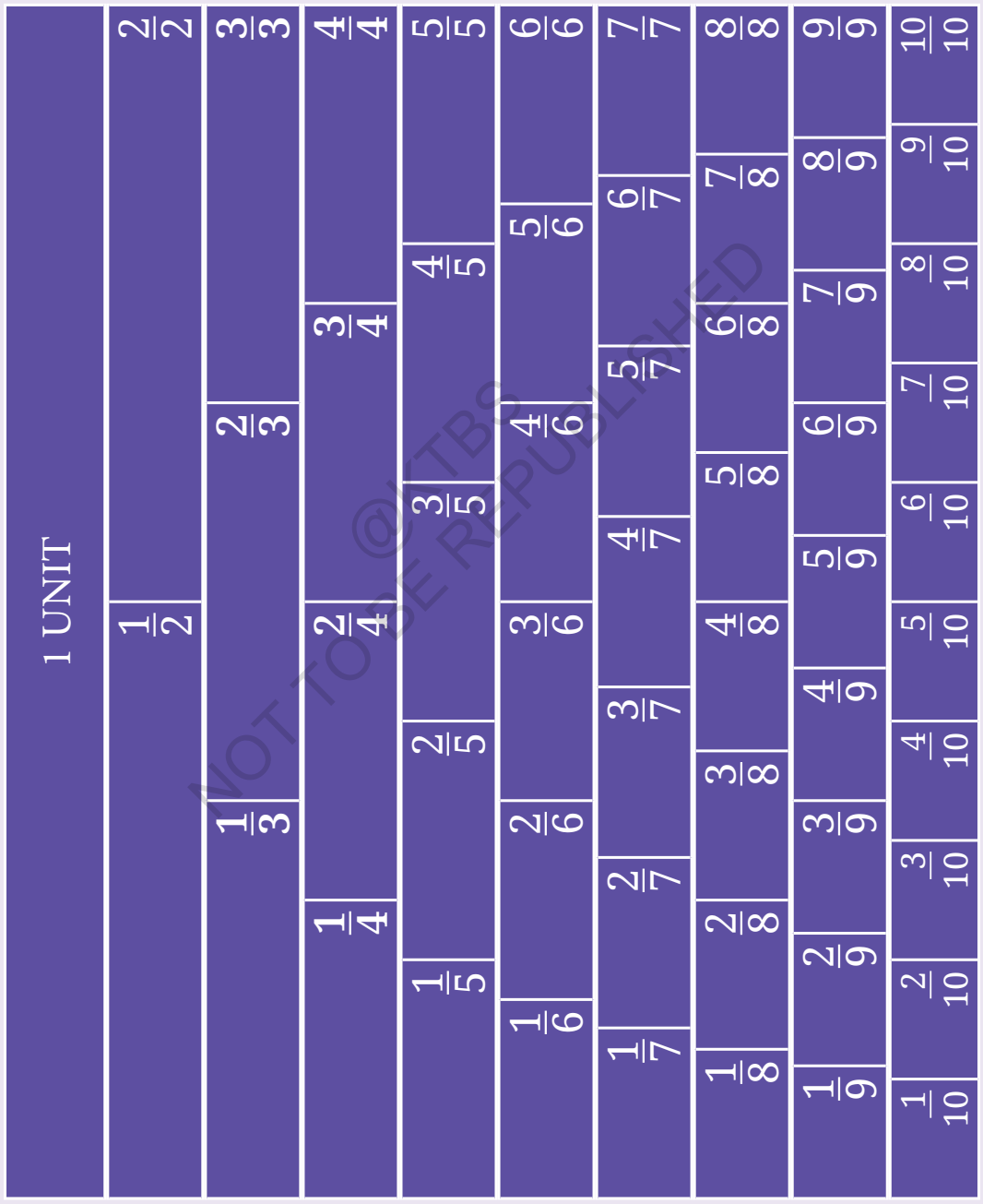


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FRACTION WALL

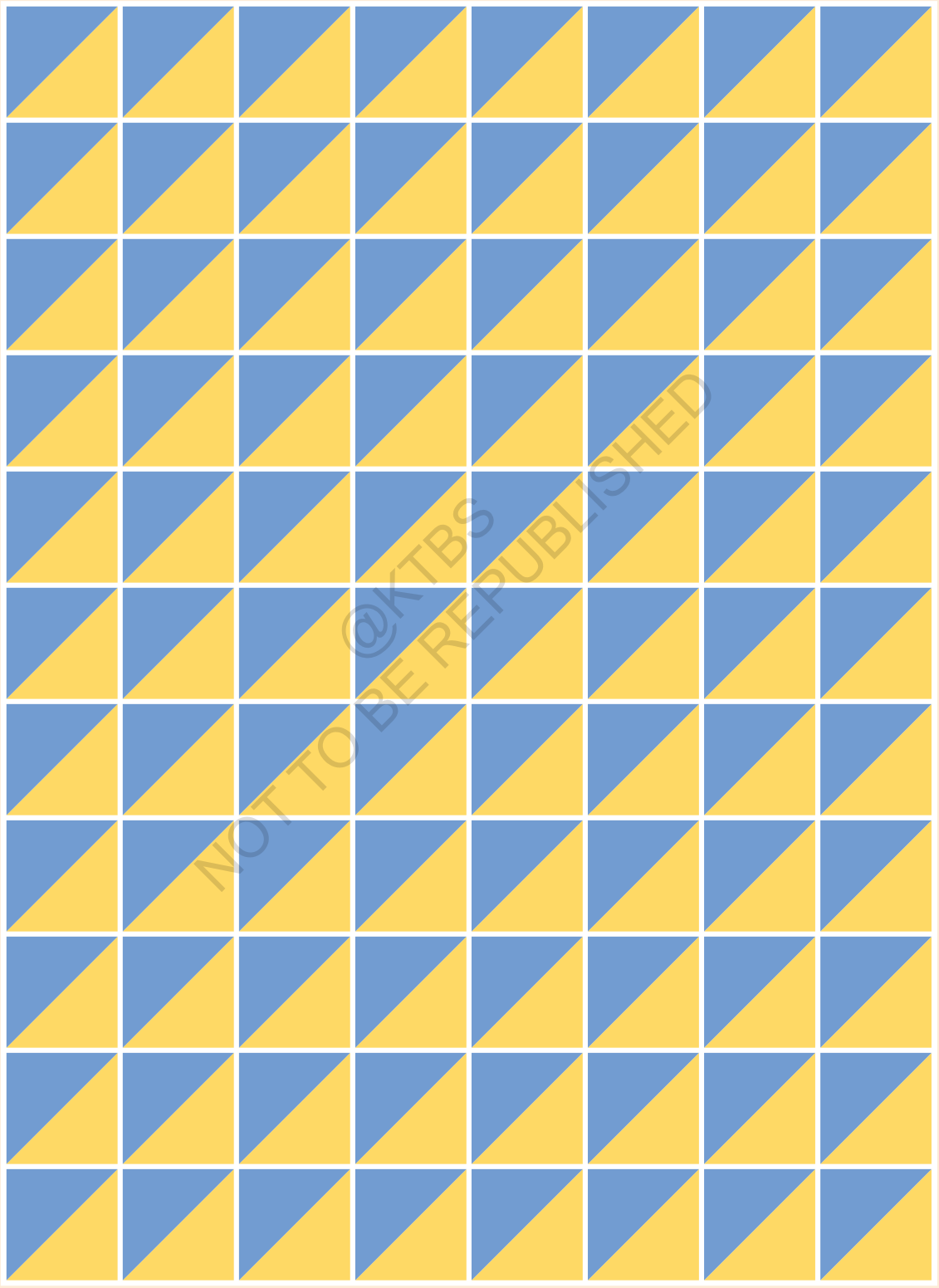
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Note: Cut the tiles along the white border.



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